JPX Working Paper【Summary】
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Investigation of Relationship between Tick Size and Trading Volume of Markets using Artificial Market Simulations

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Competition between Stock Markets

Competition Factors

Tick Size

Difficult to Change

Verification

Artificial Market Model
(Multi-Agent Simulation)

Condition Not to Move Share

$\Delta P_B > \Delta P_A$ *or* $\overline{\sigma} > \Delta P_A$

$\overline{\sigma_t}$ depend on ΔP → **Moving Share**

Mechanism of Moving Share

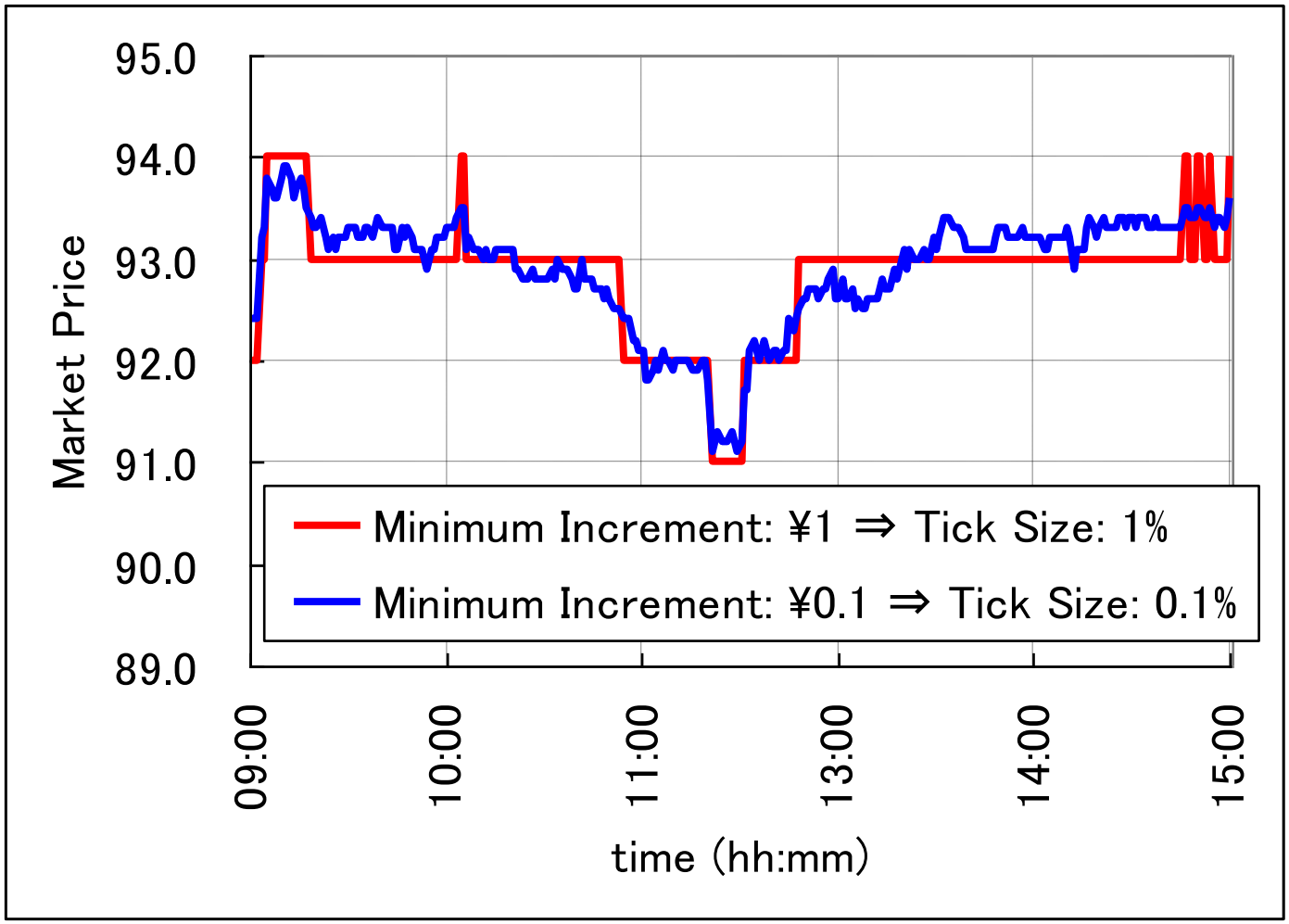
Compare

Empirical Analysis

Data: 2012 TSE and PTS

What is Tick Size?

Here, we define Tick Size ⊿P = Minimum Increment / Price



Difference of 1% Return is Serious Problem for some Investors
⇒ They prefer Stock Market has Smaller Tick Size ⊿P

Competition between Stock Markets

Competition Factors Verification

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Artificial Market Model (Multi Agent Simulation)

Chiarella et. al. [2009]

- **Continuous Double Auction**
- **Agent model is Simple**

heterogeneous 1000 agents

Expected Return

$$r_{e,j}^{t} = \frac{1}{\sum_{i=1}^{3} w_{i,j}} \left(w_{1,j} \log \frac{P_f}{P^{t-1}} + w_{2,j} r_{h,j}^{t-1} + w_{3,j} \varepsilon_j^t \right)$$

Fundamental | Technical | noise

$w_{i,j}$

Strategy Weight ↑ Different for each agent

+ Replicate Micro Structures （**Original**）

Trade number, Cancel rate, 1 day Volatility, and so on.

Simulation Time ⇔ Real Time **convertible**

We interested in how long do markets need get shares.

Terms of Fundamental Strategy, Technical Strategy

＊ Fundamental Strategy Term

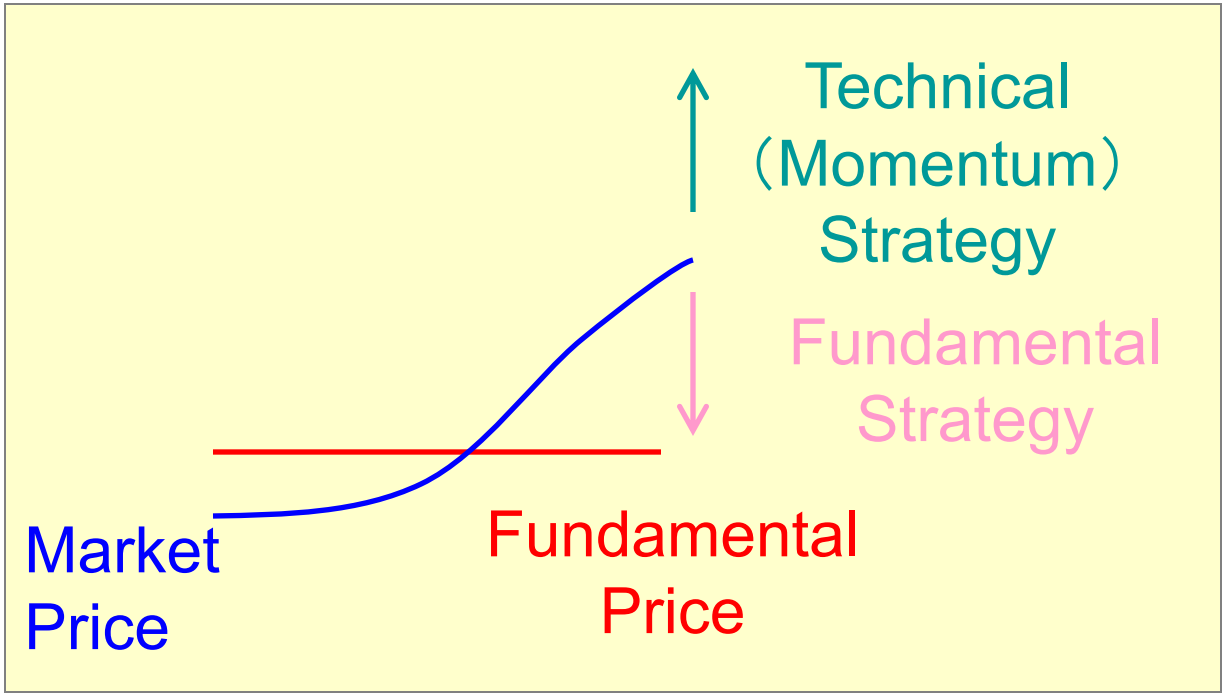
Fundamental Price ＞ Market Price ⇒ expects + return

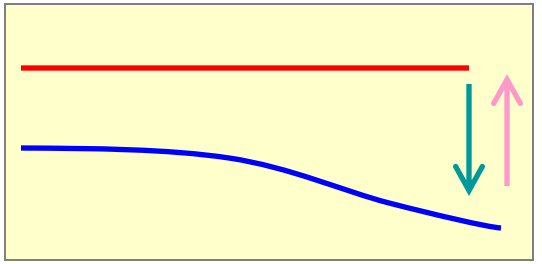
Fundamental Price ＜ Market Price ⇒ expects - return

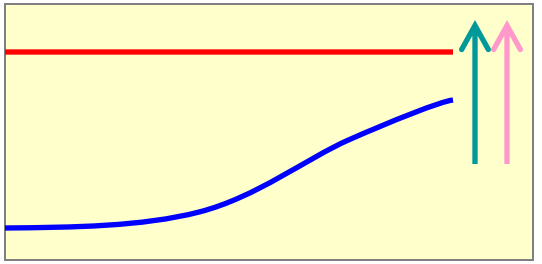
＊ Technical Strategy Term

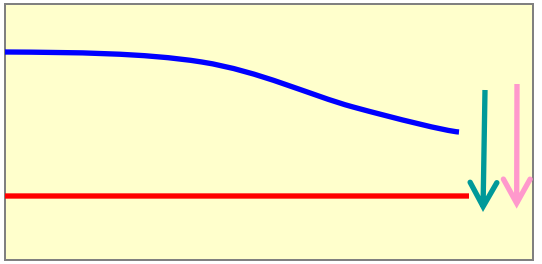
Historical Return ＞ 0 ⇒ expects + return

Historical Return ＜ 0 ⇒ expects - return

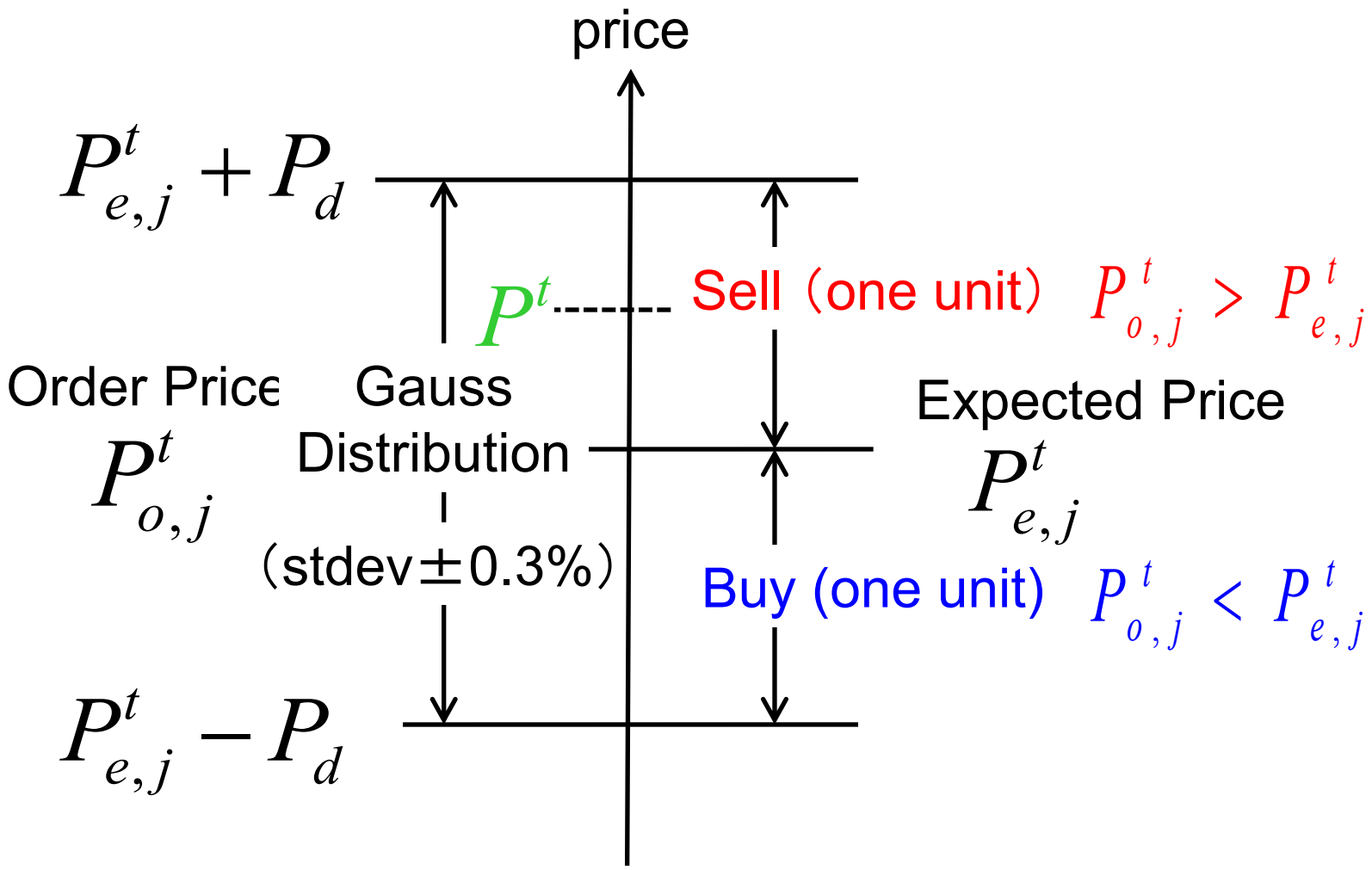








Order Price and Buy or Sell



To Stabilize simulation for continuous double mechanism, Order Prices must be covered widely in Order Book.

Agent Model Parameters

j: agent number (1000 agents)
ordering in number order

t: tick time

Historical Return

$$r_{h,j}^{t} = \log(P^{t} / P^{t-\tau_j})$$

Technical

Expected Return

$$r_{e,j}^{t} = \frac{1}{\sum_{i=1}^{3} w_{i,j}} \left(w_{1,j} \log \frac{P_f}{P^{t-1}} + w_{2,j} r_{h,j}^{t-1} + w_{3,j} \varepsilon_j^{t} \right)$$

Fundamental

noise

Parameters for agents

$w_{i,j}$ and τ_j

Random of
Uniform Distribution

$w_{i,j}$ i=1,3: 0～1
i=2: 0～10

τ_j 0～10000

P_f Fundamental Price
10000 = constant

P^t Market Price at t

ε_j^t
Random of
Normal
Distribution
Average=0
σ=3%

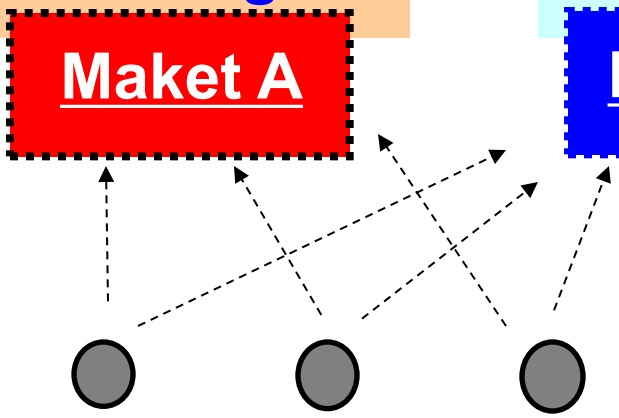
Expected Price

$$P_{e,j}^{t} = P^{t-1} \exp(r_{e,j}^{t})$$

Market Selection Model

Initial Trading Volume Share: **90%** Tick Size: **Large**

Initial Trading Volume Share: **10%** Tick Size: **Small**



Market Order: Choose the market list best price

Limit Order: Allocate on basis of Historical Trading Volume Share of each market

Market Order: buy or sell at the best available price, immediately

Limit Order: buy or sell at a specific price or better, waiting opposite Market Orders

Market Selection Model (example)

Order Book

Market A		
Sell	Price	Buy
84	101	
176	100	
	99	204
	98	77

Market B		
Sell	Price	Buy
1	99.2	
2	99.1	
	99.0	3
	98.8	1

Limit Orders

(1) Buy ¥98: Allocate on basis of Historical Trading Volume Share of each market

(2) Buy ¥99.1 : Market B

↑can buy ¥99.1 at Market B, immediately

(3) Buy ¥100 : Market B

↑can buy ¥99.1 at Market B, best price

Market B will take Trading Volume share because of (2), (3)

Allocate on basis of Historical Trading Volume Share

Wa : Probability an agent choose Market A

Ta , Tb : Trading Volume of Market A or B within last t_{AB}

t_{AB}=5 days

$$Wa = \frac{Ta}{Ta + Tb}$$

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(Multi-Agent Simulation)

Condition Not to Move Share $\Delta P_B > \Delta P_A$ ***or*** $\overline{\sigma} > \Delta P_A$


depend on ΔP → Moving Share

Mechanism of Moving Share

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Empirical Analysis

Data: 2012 TSE and PTS

Stylized Facts

	tick size(%)	0.0001%	0.001%	0.01%	0.1%	1%
about trading	trade rate	23.5%	23.5%	23.4%	23.1%	22.1%
	cancel rate	26.2%	26.2%	26.3%	26.6%	27.6%
	number of trades / 1 day	6,361	6,358	6,345	6,279	6,081
standard deviations	for 1 tick	0.05%	0.05%	0.05%	0.06%	0.16%
	for 1 day (20000 ticks)	0.59%	0.56%	0.57%	0.57%	1.15%
kurtosis		1.50	1.48	1.45	1.10	1.81
autocorrelation coefficient for square return	lag					
	1	0.229	0.228	0.228	0.210	0.025
	2	0.141	0.141	0.141	0.120	0.013
	3	0.109	0.108	0.108	0.090	0.008
	4	0.091	0.091	0.091	0.075	0.006
	5	0.078	0.078	0.078	0.064	0.004

Replicate Fat-Tail and Volatility-Clustering

$\bar{\sigma}_t = 0.05\%$

Volatility at tick size small

\+ Replicate Micro Structures (**Original**)

Trade rate, Cancel rate, 1 tick and 1 day volatility

Simulation Time ⇔ Real Time **convertible**

We interested in how long do markets need get shares.

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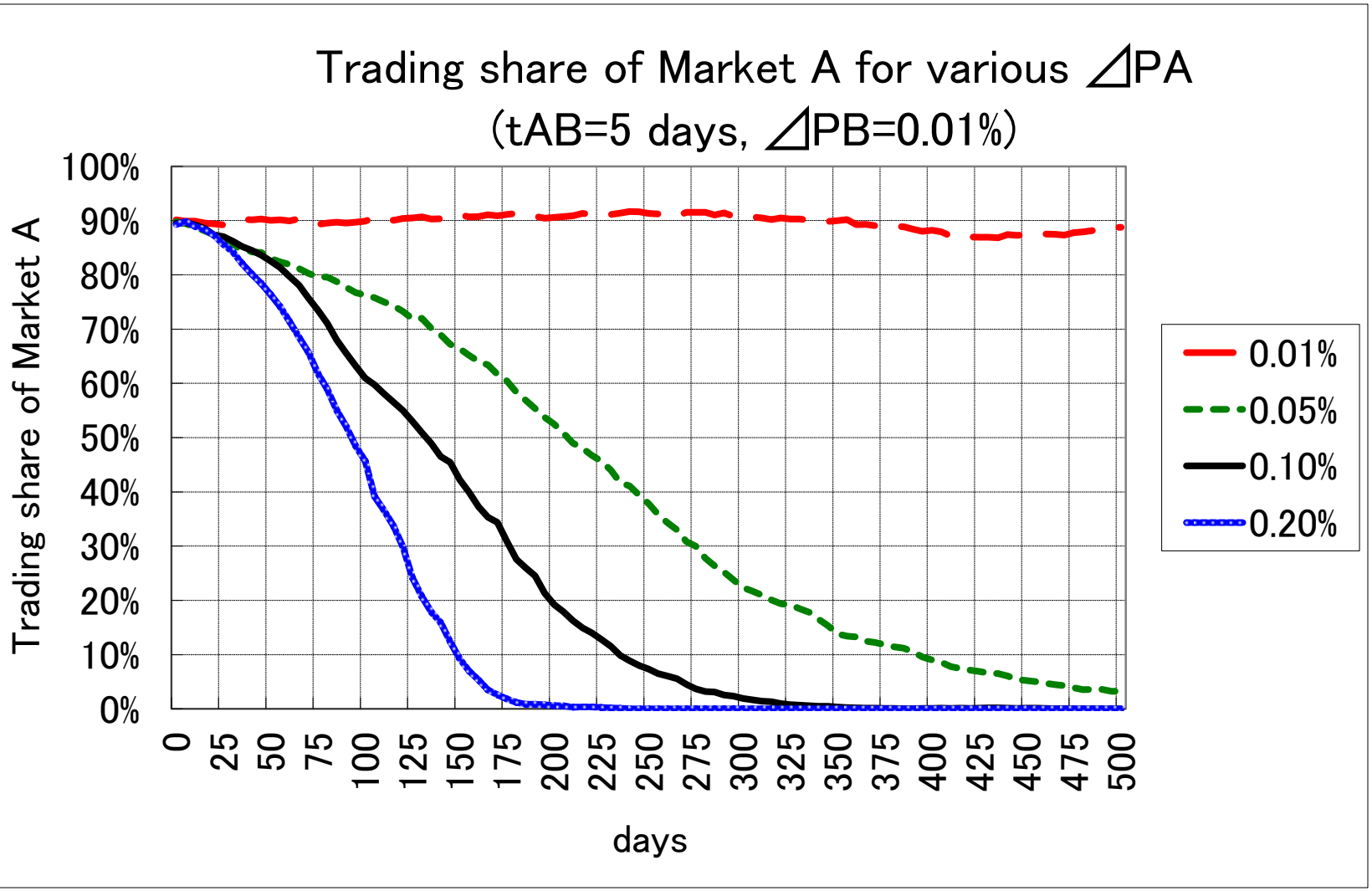
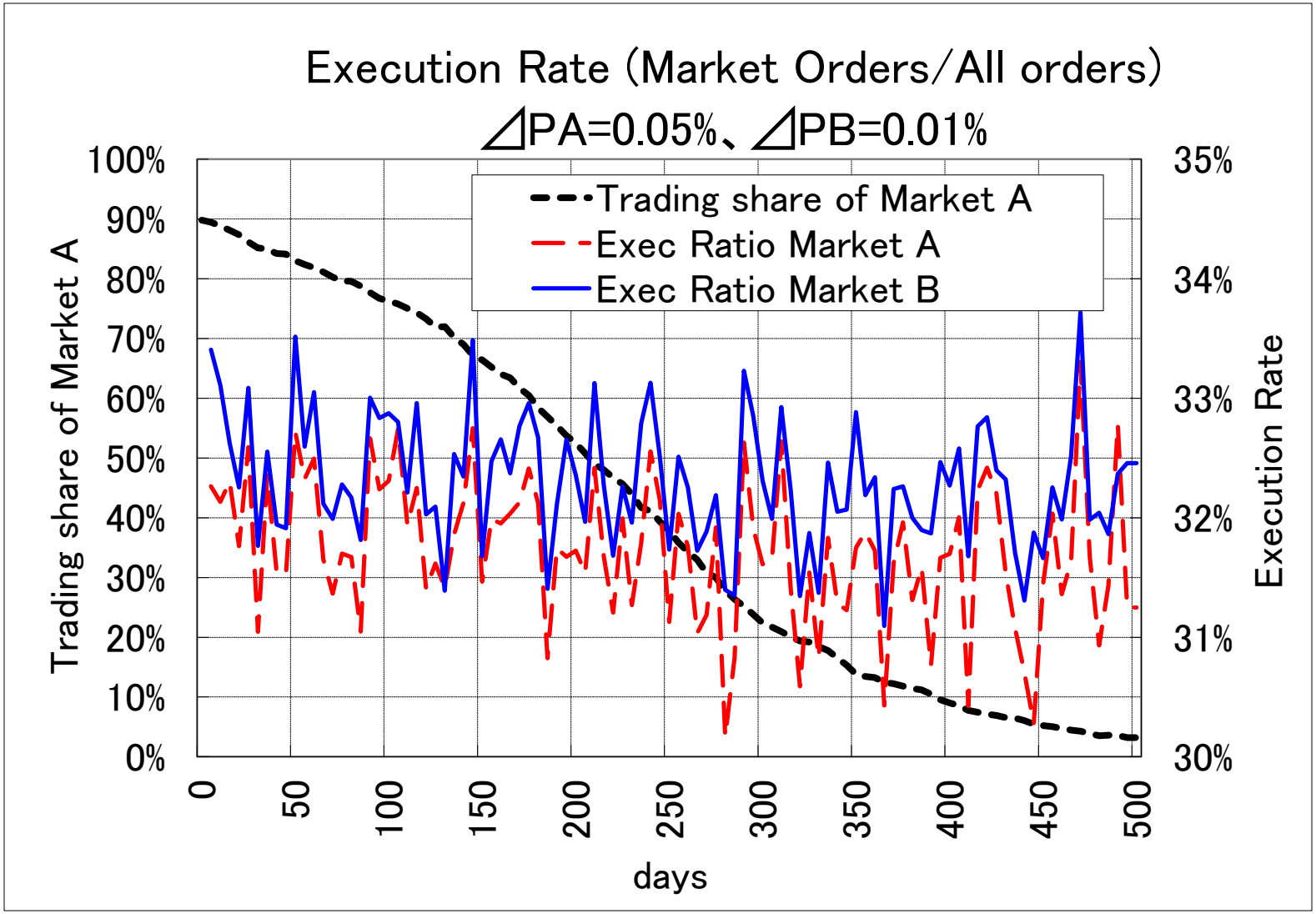
Tick Size of Market B ⊿PB=0.01%, Tick Size is not small



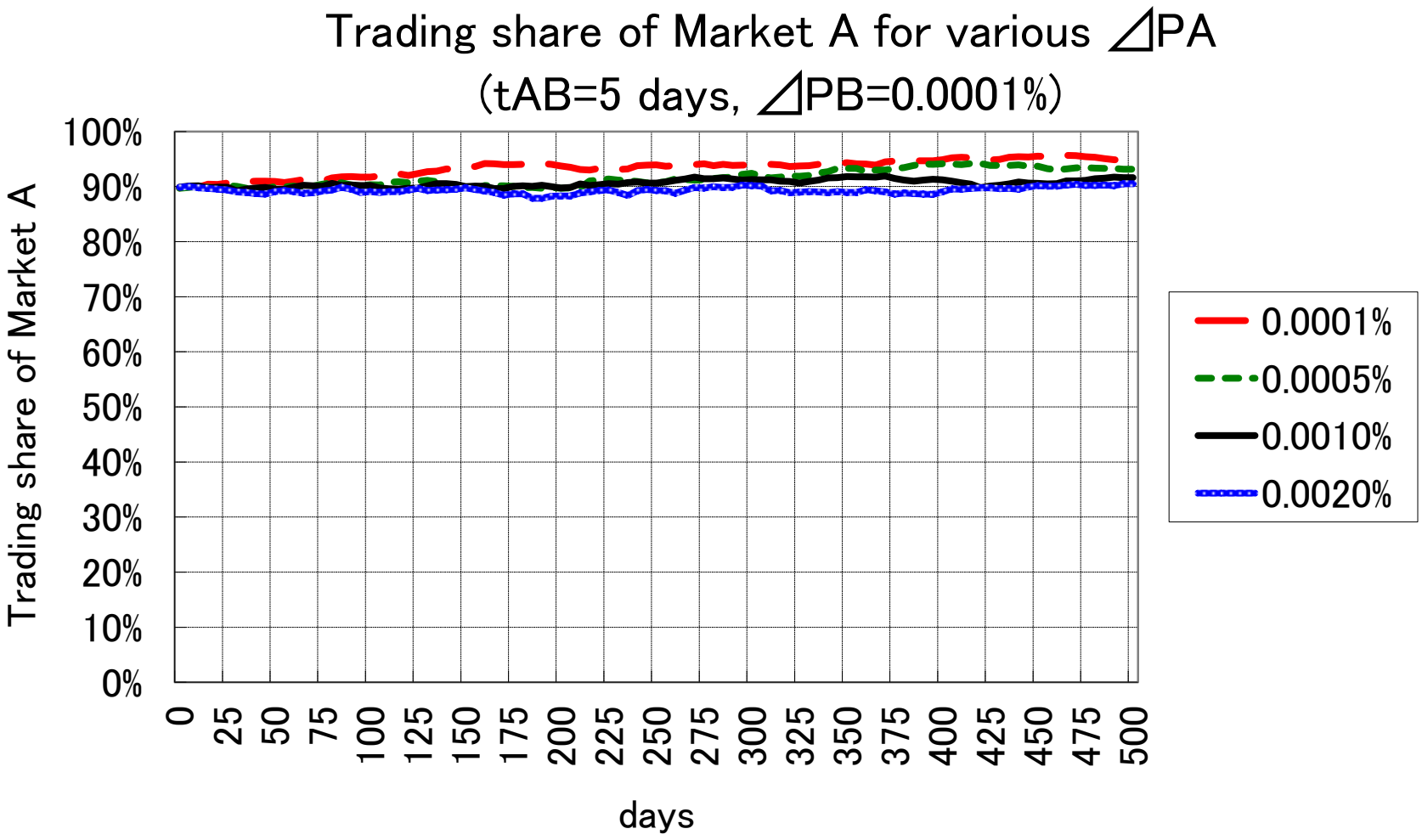
Tick Size of Market A, ⊿PA is larger,
Market A is taken trading volume share faster

Executions Rate (Market Orders/All Orders)



Execution Rate of Market B was slightly bigger than that of Market A. Because of the difference, Market B took the share

⊿PB=0.0001%, Tick Size is enough small



Market B can hardly take the share in spite that ⊿PA is very larger than ⊿PB

Tick Size Condition Not to Move Share

Trading share of Market A at 500 days		⊿PB										
		0.0001%	0.0002%	0.0005%	0.001%	0.002%	0.005%	0.01%	0.02%	0.05%	0.1%	0.2%
⊿PA	0.0001%	90%	90%	91%	91%	92%	94%	97%	99%	100%	100%	100%
	0.0002%	90%	90%	90%	91%	91%	94%	97%	99%	100%	100%	100%
	0.0005%	89%	90%	91%	91%	92%	94%	96%	99%	100%	100%	100%
	0.001%	89%	89%	90%	90%	92%	94%	97%	99%	100%	100%	100%
	0.002%	87%	88%	89%	89%	91%	93%	97%	99%	100%	100%	100%
	0.005%	84%	85%	85%	84%	87%	92%	96%	99%	100%	100%	100%
	0.01%	75%	76%	76%	77%	78%	83%	92%	98%	100%	100%	100%
	0.02%	53%	52%	53%	54%	54%	59%	70%	93%	100%	100%	100%
	0.05%	5%	5%	4%	5%	5%	5%	6%	23%	93%	100%	100%
	0.1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	94%	100%
	0.2%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	96%

Condition Not to Move Share $\Delta P_B > \Delta P_A$ *or* $\bar{\sigma} > \Delta P_A$

$$\bar{\sigma}_t = 0.05\%$$

Competition between Stock Markets

Competition Factors Verification

Tick Size

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Condition Not to Move Share

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$\overline{\sigma}_t$ depend on ΔP

Moving Share

Mechanism of Moving Share

Compare

Empirical Analysis

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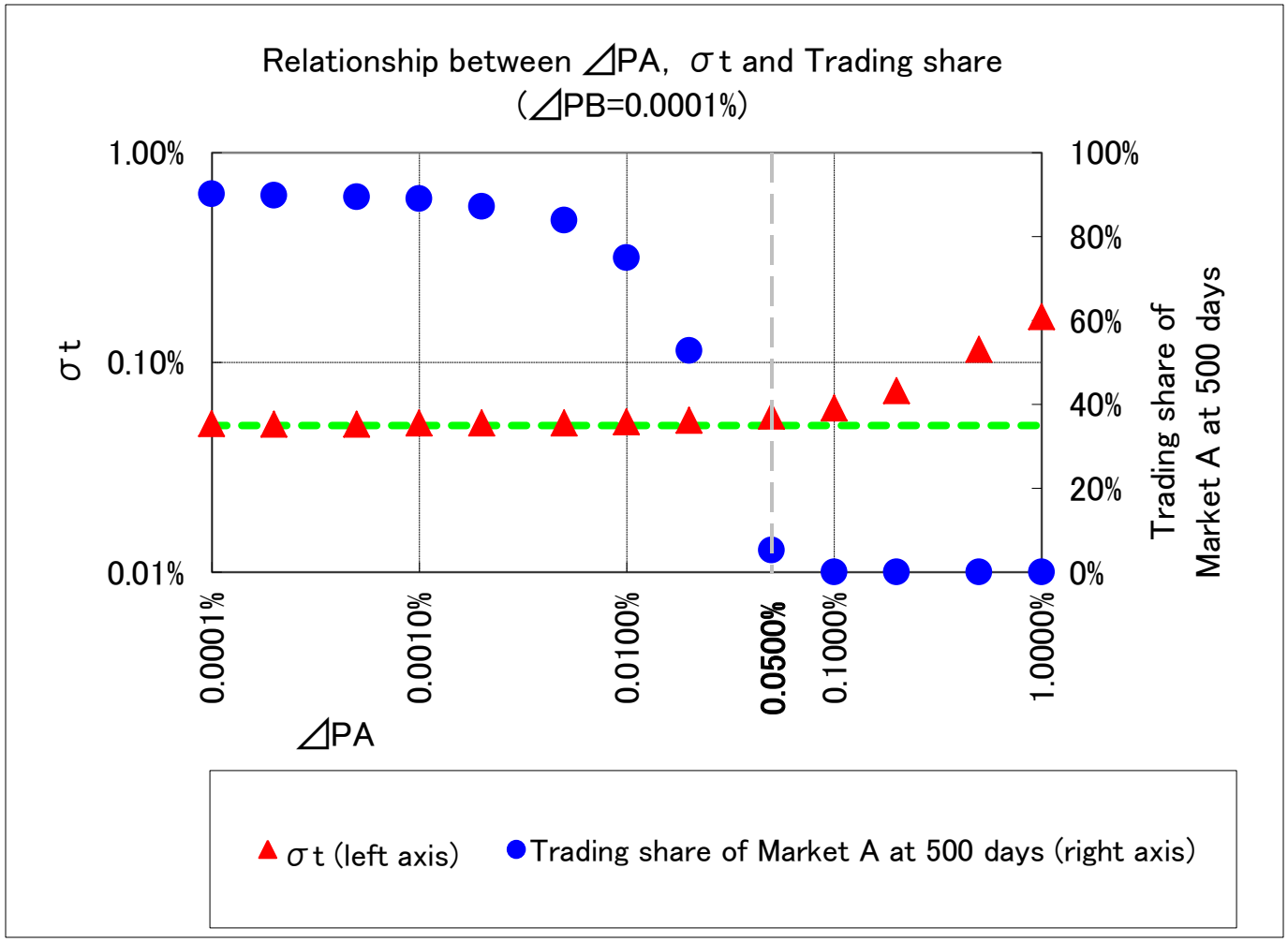
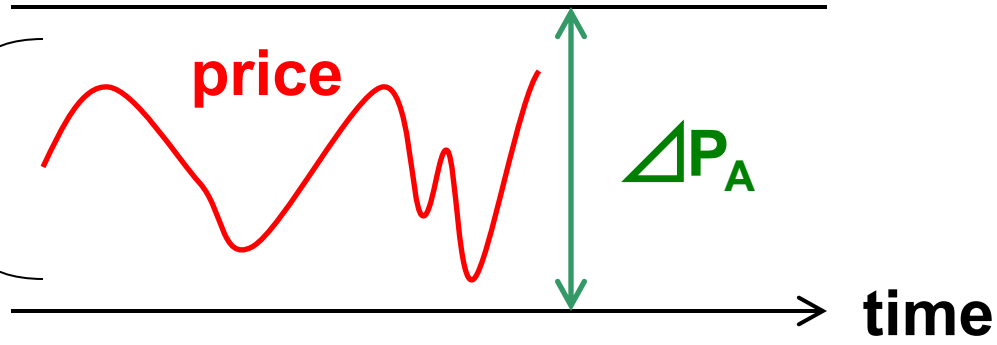
Relationship between σt and Share (⊿PB is enough small)



When σt depends on ⊿PA, Market A is taken share very Rapidly

$\overline{\sigma_t} < \overline{\Delta P_A}$

unable trading in Market A



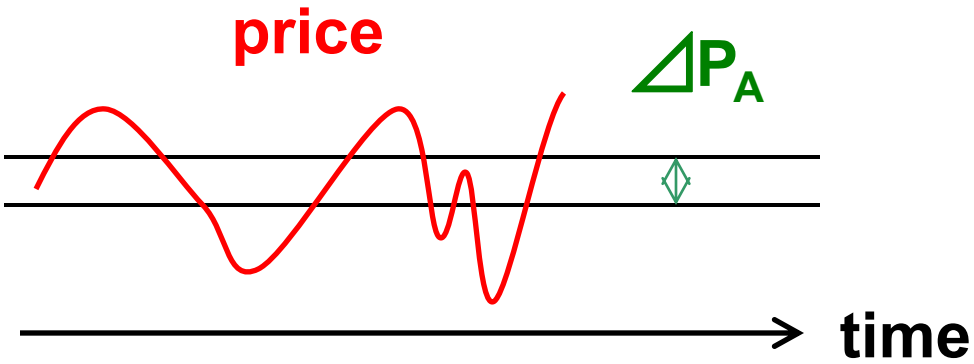
unable trading in Market A

→ many trading in Market B

⇒ trading share moving to Market B

$\sigma_t > \overline{\Delta P_A}$

needless Market B



⇒ trading share not moving

Competition between Stock Markets

Competition Factors Verification

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(Multi-Agent Simulation)

Condition Not to Move Share $\Delta P_B > \Delta P_A$ *or* $1/10 > \Delta P_A$

$\overline{\sigma}_t$ depend on ΔP → Moving Share

Mechanism of Moving Share

Compare

Empirical Analysis

Data: 2012 TSE and PTS

Empirical Study

Data

Data Period: All business days in calendar year 2012

Universe: 439 stocks

Selected by TOPIX 500 index whole data period
they had same tick size for every month ends
they were traded every business days at least once

Horizontal Axis: Tick Size of TSE ⊿P for each stock

▲: standard deviation of 10 seconds return for each stock, σt

●: trading volume share in PTS for each stock

Summarize Markets:

Traditional Stock Exchanges:

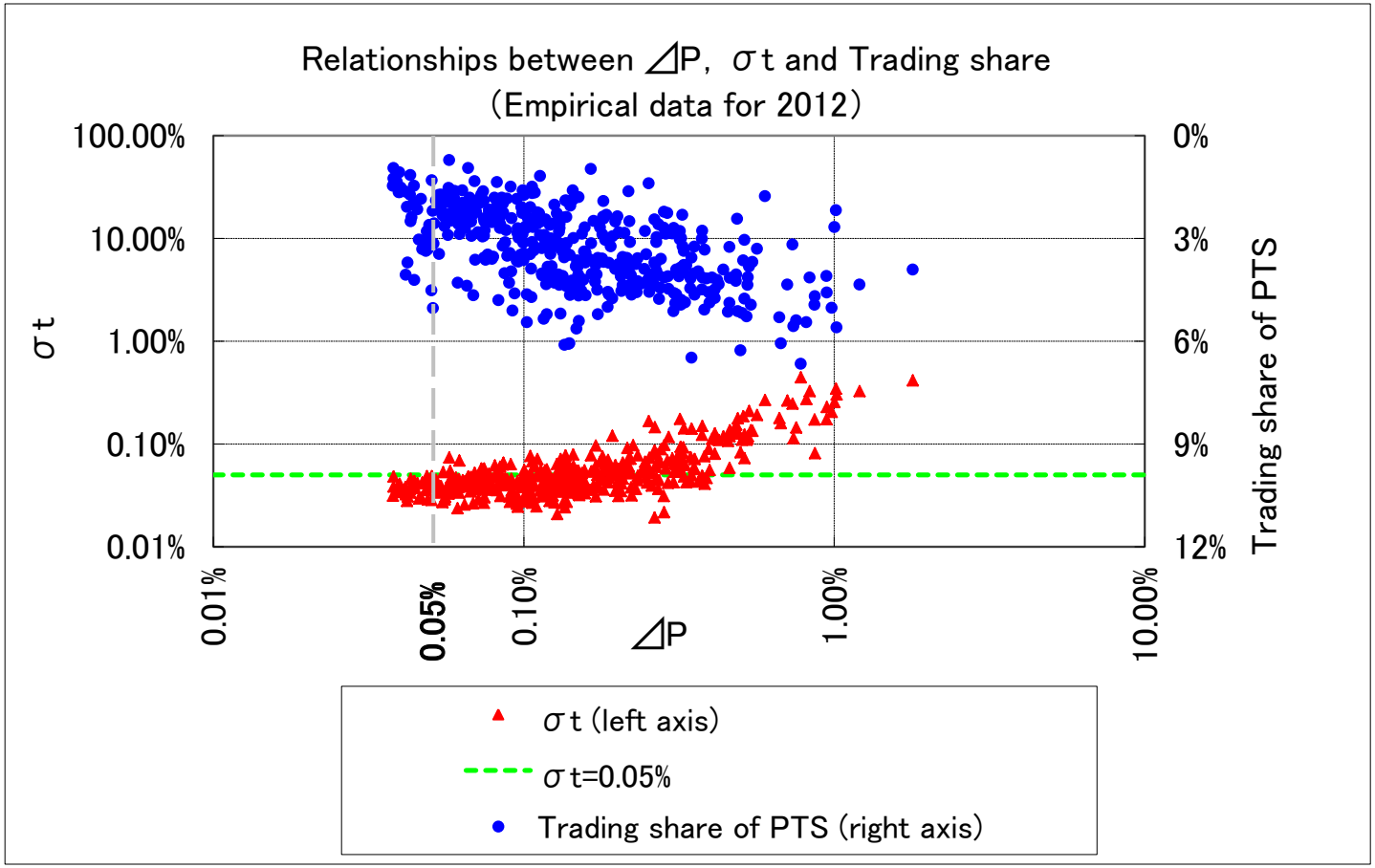
Tokyo Stock Exchange, Osaka SE,
Nagoya, Fukuoka, Sapporo, and JASDAQ

PTS (Proprietary Trading System):

Japan Next PTS J-Market, Japan Next PTS X-Market,
and Chi-X Japan PTS

Empirical Result

(right vertical axis is reversed)



Right Side, Volatility σt depends on Tick Size ⊿P, Tokyo Stock Exchange is taken share more.

Summary

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Appendix

Definition of Market/Limit order

In this study

A little difference from actual market

All agents decide an order price

sell

order book

buy

sell	price	buy
84	101	
176	100	
	99	2
	98	77

limit

market

Exist matching order
Order executed immediately

market

limit

No matching order
Order not executed immediately

Agents decide an order price,
if exist matching order, market order else limit order