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Analysis of Short Side Market Inefficiencies

Using Artificial Market Model

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Overview

We used an artificial market model to analyze the factors contributing to the inefficiencies on the short side (overvaluation) of stock market contract prices exceeding fundamental prices. Differences in purchase and sell order quantities issued by agents, with a smaller sell volume than purchase volume, caused a short side breakdown of market inefficiency to exceed that of the long side (undervalued). However, we were unable to confirm that when the fundamental price used to determine the agent's order price increased moderately, the breakdown on the short side exceeded that on the long side. We discuss the mechanism by which market inefficiency on the short side increases, where the best offer price deviates from the fundamental price because the volume of sell orders presented to the board within a certain range from the fundamental price is smaller than that of purchase orders, thereby increasing the market impact of the purchase orders. In an empirical analysis using Tokyo Stock Exchange (TSE) order data, we calculated the ratio of sell to purchase orders as part of the ordered quantity. The median of the sell/purchase ratio was ~1 in a group of stocks belonging to the TSE First Section, whereas that for stocks in other market segments tended to be <1.

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1. Introduction

Although price inefficiencies are positioned as an alpha[1] source for market investment, more inefficiency may exist on the short side than long side.

One hypothesis proposed for this is that impediments to shorting stocks result in less investment in overvalued (shorting) stocks than in undervalued stocks[1]. For example, shares are borrowed for short selling via lending market[2], and there are various restrictions on short selling. These include a shortage of stock available for lending and high procurement cost. In this study, we hypothesize that the existence of short-selling constraints makes it difficult for concerns about falling prices to be reflected in actual investments.

Another hypothesis is the content of published information may be biased. For example, [2] focused on portfolios that gained a return using historical US company data regarding disclosures to measure time-series similarity, shorting company groups wherein the account-reporting terminology and structure changed, and purchasing company groups wherein the account-reporting terminology and structure did not change. Additionally, [3] demonstrated survey results indicating that security companies (sell side analysts) issue many more reports with buy recommendations than sell recommendations[3]. This could be because people in management and analysts are more reluctant or rigid when disclosing negative information than positive information, which is proactively disclosed to investors[1].

However, investigating the impact of individual changes and mechanisms in terms of the aforementioned factors through empirical research alone is challenging because other factors may contribute to price formation. Therefore, in this study, we propose an artificial market simulation method that uses a computer to create an agent-based model of the securities market. Hereafter, models using artificial market simulation are referred to as artificial market models.

The artificial market model employs multiple agents that mimic investors and an exchange that mimics the same. The simulation is entirely computer-based[4]. We modeled a situation in which investors have to reduce the number of sell orders due to short-selling constraints and a situation in which additional positive information is published. We could compare the simulated executed price and the fundamental price used to determine the agent's order price by adjusting the factors

[1] This refers to the excess return obtained from an investment.

[2] The reason for this is that naked short selling (short selling in which shares are not allocated at the time of sale) is not allowed in Japan.

[3] It was also indicated that the market response is asymmetrical in the case of sell recommendations and buy recommendations.

[4] Note that this analysis is independent of the actual stock market.

(parameters) separately, allowing us to examine whether the short side of the price deviation (market inefficiency)[5], i.e., the breakdown of the overvalued side above the fundamental price, increases at the end of the transaction. Following validation, we examine the mechanism underlying the short-side market inefficiency.

In this study, the volume ratio of sell to purchase orders is calculated using the empirical analysis of TSE order data. Further, we examine the relationship between short-selling constraints (with stock certificates supplied to the lending market) and artificial market model parameters.

[5] Market inefficiency can directly be calculated by using the artificial market model[6].

2. Artificial Market Model

Based on [6], we constructed a simple model within the scope of the study while satisfying the analysis results. According to [4], reports indicate that complex agent-based models do not often increase the number of stylized factors (the types of statistical properties found in real markets) that can be reproduced. Additionally, a complex model with additional parameters will make the analysis difficult, this study does not aim to perfectly reproduce the actual market. Simultaneously, this study assumes normal market trading; therefore, the parameters[6] to be changed individually in the simulation were determined in such a way that they do not deviate substantially from the original values. The reason for this is that the method for implementing the artificial market model in this study is the continuous double auction (so-called "zaraba") method, as described below, and if the simulation results in frequent and abrupt price fluctuations because of the extreme parameters, the usual zaraba method alone cannot reproduce the price determination mechanism[7].

Although several studies investigate artificial market simulation focusing on short selling, such as [7], we did not come across any study that focus on the inefficiencies due to the long and short sides asymmetry, where the execution price deviates from the fundamental price; the present study attempts to fulfill this knowledge gap. In addition, in terms of how the fundamental price can be changed to determine the agent's order price, there are studies such as [7] that show a sharp drop at a certain point in time. However, no study has looked at more gradual changes, such as the information published under normal circumstances.

2.1. Transaction Process

The model is based on a single stock[8], and the pricing mechanism is a zaraba system in which the offer price is matched between the seller's and the purchaser's orders, and the same quantity of orders are executed immediately at a specific price.

This involves preparing n agents as investors, starting from agent $j = 1$, and placing orders $j = 1,2,3, ...$, consecutively. When the orders are placed up to agent $j = n$, they are repeated from the first agent $j = 1$ the next time. For each order placed by a single agent, $t = 1,2,3, ...$ However, time progresses even when the order is just placed and not executed.

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[6] These are the parameters q and d described later.
[7] It is also considered that making the model more complex, for example, by setting up a special quote mechanism in the real stock market is necessary.
[8] We assume that there is no change in the stock price by corporate actions.

Agent j determines the order price and whether to purchase or sell as follows. The price change rate (predicted return) $r_{e,j}^t$ predicted by agent j at time t is

$$r_{e,j}^t = \frac{w_{1,j} \log \frac{P_f^t}{P^{t-1}} + w_{2,j} r_{\hbar,j}^t + u_j \epsilon_j^t}{w_{1,j} + w_{2,j} + u_j} \tag{1}$$

Here, $w_{i,j}$ is the weight of the i^{th} item of agent j, and is determined as a uniform random variable from 0 to $w_{i,max}$ when determining the random variable table to be referenced in the simulation. Additionally, u_j is the weight of the third item of agent j, and is determined as a uniform random variable from 0 to u_{max}. The term log refers to a natural logarithm. P_f^t refers to the fundamental price at time t, and P^{t-1} is the executed price at time $t-1$ (if there is no executed order at time $t-1$, uses the price at the closest time. When $t = 1$, $P^{t-1} = P_f^1$[9]. ϵ_j^t is the disturbance term at time t, agent j, and is a random variable that follows a nominal distribution with mean 0 and standard deviation σ_ϵ. $r_{\hbar,j}^t$ is the past return calculated by agent j at time t, and is $r_{\hbar,j}^t = \log(P^{t-1}/P^{t-\tau_j-1})$. Here τ_j is determined as a uniform random variable from1 to τ_{max} for each agent.

The first term in numerator of Equation $(1)(1)(1)(1)$ refers to the fundamental investor who makes investment decisions by calculating the difference between the contract and fundamental prices. If P^{t-1} is cheap (expensive) compared to the fundamental price, a positive (negative) prediction return is expected. The second term refers to the technical investor, who considers past price movements while making investment decisions. If $r_{\hbar,j}^t$ is positive (negative), the predicted return is expected to be positive (negative). The third term expresses noise.

The predicted price $P_{e,j}^t$ is obtained as

$$P_{e,j}^t = P^{t-1} \exp\left(r_{e,j}^t\right) \tag{2}$$

Order price, $P_{o,j}^t$ is a random variable following the nominal distribution of mean $P_{e,j}^t$, and the standard deviation P_σ^t. P_σ^t is obtained from Equation $(3)(3)$. Note that σ here is a time-dependent constant.

$$P_\sigma^t = P_{e,j}^t \, \sigma \tag{3}$$

The difference between purchasing and selling is determined by the relationship between the predicted price $P_{e,j}^t$ and the order price $P_{o,j}^t$, which is determined as follows:

$$\begin{gathered} P_{e,j}^t > P_{o,j}^t \text{ then purchase} \\ P_{e,j}^t < P_{o,j}^t \text{ then sell} \end{gathered} \tag{4}$$

9 $P^{t-\tau_j-1}$ described later is treated in the same way.

(However, time $t \leq 20{,}000$ is the board building period, and regardless of Equation (4), if $P_{o,j}^{t} > P_f^1$ then sell; Alternatively, if $P_{o,j}^{t} < P_f^1$, then purchase). The smallest order price unit (tick size) is δP, with adjustments for smaller fractions[10].

With this model, prices are formed in a zaraba format. Therefore, if there are sell (purchase) orders on the board lower (higher) than the price quoted by the purchase (sell) order, they are executed sequentially according to the principle of price priority. P^t is treated as the price at which an order is executed at time t[11]. Orders that are not executed remain on the board. If the order remains on the board and is not executed after a lapse of time t_c, it is canceled. Any agent may submit orders in sequence $(j = 1, \ldots, n)$ as many times as they wish (infinite cache) until time t_e, which is the time at which trades are closed.

2.2. Situation in which there are constraints on sell orders

In this study, we could simulate a situation in which the number of sell orders is less than the number of purchase orders on average, which was not possible in [6]. This is possible because while there are no limitations on placing a purchase order by unlimited cash, investors are not as free to place a sell order because of the need to procure shares for a short sale and other restrictions that may exist, such as a shortage of stock available for lending and higher procurement costs. We can generalize this situation as a simulation in which the artificial market model has a low volume of sell orders relative to purchase orders.

More specifically, the purchase order quantity is set to 100, and the sell order quantity is set according to the parameter q in Equation (5)[12].

$$q = \{100,99,98,95,90\} \tag{5}$$

If $q = 100$, there is no difference in order quantity; however, as q decreases, only the quantity of sell orders decreases. In other words, the parameter q represents how much the total volume of sell orders placed in the market will decrease. From the beginning of the transaction (one simulation) to the end, the value of q is assumed to be constant.

2.3. Situation in which More Information with Positive Content is Published

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[10] Sell orders are rounded up, and purchase orders are rounded down.

[11] When an order is executed with several orders on the opposite side of the board, the price executed in the end applies to P^t.

[12] The order price $P_{o,j}^{t}$ is the same price each time.

Another difference between this study and [6] is the ability to create a simulation in which the fundamental price used by the agent to determine the order price gradually increases over time[13]. More specifically, fundamental price P_f^t is defined as in Equation (6)(6),

$$P_f^t = \begin{cases} P_f^1 & (t \leq \text{board building period}) \\ P_f^1 \left(1 + d \times \dfrac{t - \text{board building period}}{t_e - \text{board building period}}\right) & (t > \text{board building period}) \end{cases} \tag{6}$$

and the increase rate in the fundamental price at the time t_e is set to parameter d (%) as shown in Equation (7).

$$d = \{\pm 0, +1, +2, +5, +10\} \tag{7}$$

If $d = \pm 0$, the content of information published is equal and the fundamental price is constant; however, as d increases, more information with more positive than negative content is published. If $d > 0$, the fundamental price will increase over time, and the level of the agent's expected price will increase simultaneously.

2.4. Indicators for Measuring the Asymmetry of Market Inefficiencies

In this study, the short and long sides of the divergence between fundamental and execution prices are directly measured using the indicator M_{ie}.

$$M_{ie} = \frac{1}{t_e} \sum_{t=1}^{t_e} \frac{|P^t - P_f|}{P_f} \tag{8}$$

For market inefficiencies reported in [6], we define a new market inefficiency $M_{ie,ov}$ when the execution price P^t at time t[14] is higher than the fundamental price (short side) and conversely, market inefficiency $M_{ie,uv}$ when P^t is lower than the fundamental price (long side).

$$\begin{aligned} M_{ie,ov} &= \sum_{\substack{1 \leq t \leq t_e \\ P^t > P_f^t}} \frac{|P^t - P_f^t|}{P_f^t} \\ M_{ie,uv} &= \sum_{\substack{1 \leq t \leq t_e \\ P^t < P_f^t}} \frac{|P^t - P_f^t|}{P_f^t} \end{aligned} \tag{9}$$

|| represent absolute values. If $d > 0$, the midway increase in the fundamental price is excluded from $|P^t - P_f^t|$. $M_{ie,ov}$ and $M_{ie,uv}$, like M_{ie}, take values greater than or equal to 0, indicating that they are perfectly efficient if 0 and grow more inefficient as the value increases.

[13] From [2] and [3], we modeled the market in this way, assuming that fundamental prices are almost constant or slightly rising in the normal market assumed in the simulations in this study.

[14] When there are no executions at time t, P^t refers to the latest execution price.

If inefficiencies are confirmed, $\log(M_{ie,ov}/M_{ie,uv})$ is calculated, and if the result is greater than 0, the breakdown on the short side is larger; alternatively, if it is lower than 0, the breakdown on the long side is larger.

3. Simulation Results

In this study, the parameters of the artificial market model are set as $n = 1{,}000, w_{1,max} = 1, w_{2,max} = 10, u_{max} = 1, \sigma_{\epsilon} = 0.06, \tau_{max} = 10{,}000, \sigma = 0.003, t_c = 20{,}000, \delta P = 10, P_f^1 = 10{,}000{,}000$ and $t_e = 5{,}000{,}000$ (each agent can order up to 5,000 times).

Furthermore, two patterns (assuming all other conditions are equal) were simulated: one in which $d = \pm 0$ is fixed and q is varied, as in Equation (5) in the situation where there are constraints on sell orders, and one in which $q = 100$ and d is varied as in Equation (7) in the situation where more positive information is disclosed (all other conditions being equal).

This process was then repeated 50 times, changing the random variable table to stabilize the results. After the simulation, the mean value of $\log(M_{ie,ov}/M_{ie,uv})$ was obtained for each pattern (all results revealing that there were market inefficiencies on both the short and long sides).

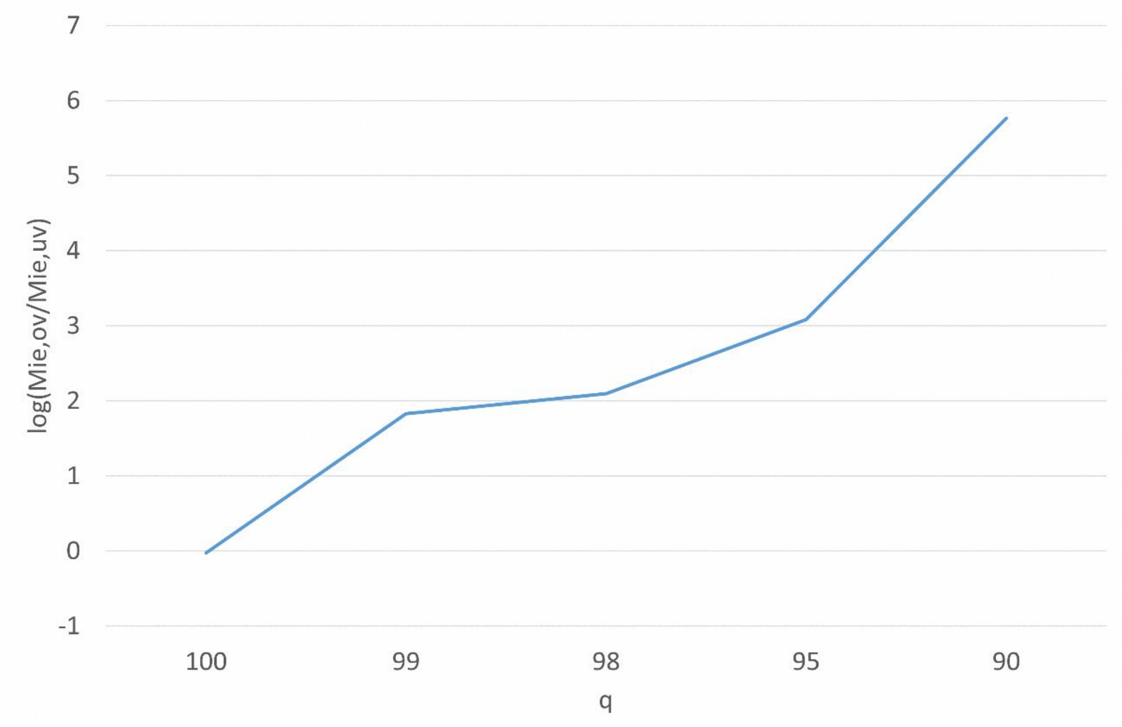

Figure 1 $\log(M_{ie,ov}/M_{ie,uv})$ **for each** q

3.1. $\log(M_{ie,ov}/M_{ie,uv})$ when changing q

Table 1 $\log(M_{ie,ov}/M_{ie,uv})$ **for each** q

q	100	99	98	95	90
$\log(M_{ie,ov}/M_{ie,uv})$	-0.027	1.827	2.095	3.082	5.765

Table 1 lists the $\log(M_{ie,ov}/M_{ie,uv})$ for each parameter q (Figure 1 is a line graph showing the same results). If $q = 100$, there is no significant difference in the long and short sides breakdown in market inefficiencies, but as q decreases, $\log(M_{ie,ov}/M_{ie,uv})$ increases[15]. Based on the above results, a decrease in the sell order volume placed in the market reveals market inefficiency on the short side.

3.2. $\log(M_{ie,ov}/M_{ie,uv})$ when d is changed

Table 2 $\log(M_{ie,ov}/M_{ie,uv})$ **for each** d

d	± 0	+1	+2	+5	+10
$\log(M_{ie,ov}/M_{ie,uv})$	-0.027	−0.033	−0.036	−0.054	−0.063

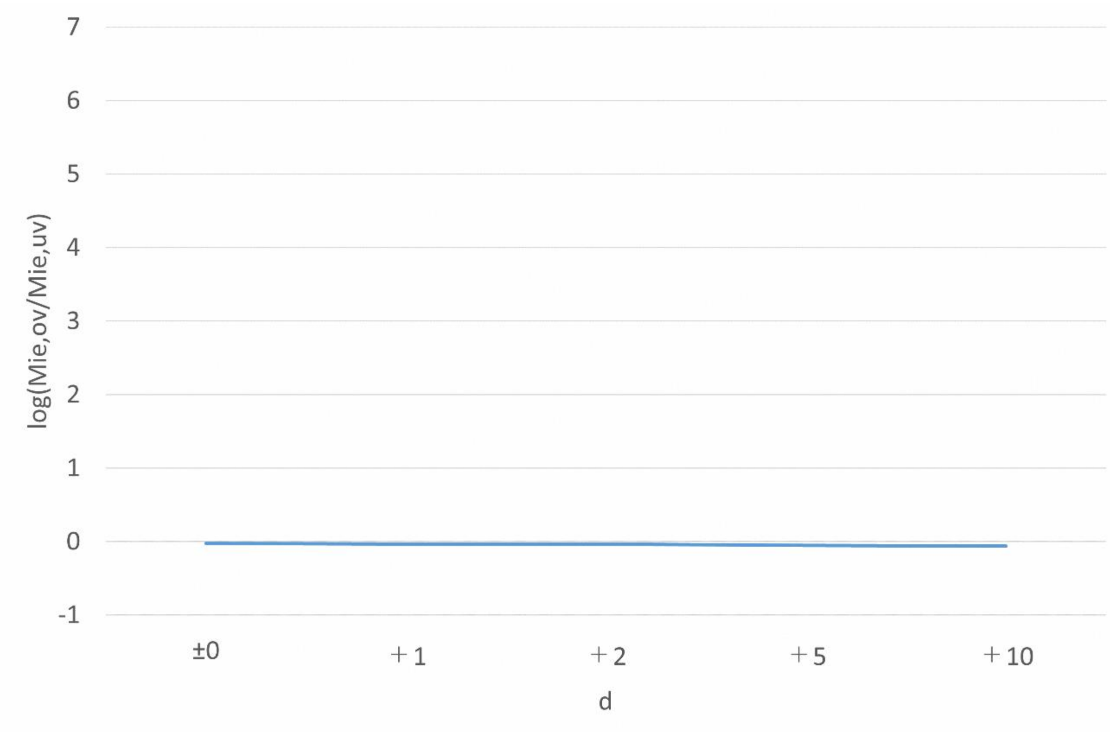

Figure 2 $\log(M_{ie,ov}/M_{ie,uv})$ **for each** d

[15] See Appendix B for the transition in the time series.

Table 2 shows the $\log\left(M_{ie,ov}/M_{ie,uv}\right)$ for each parameter d (Figure 2 is a line graph showing the same results). If $d = \pm 0$, there is no significant difference between the short and long sides breakdown in terms of market inefficiency, and the short-side breakdown does not increase as d increases (according to the results of Table 2, as d increases, $\log\left(M_{ie,ov}/M_{ie,uv}\right)$ seems to get slightly small). We could not confirm from the above whether a gradual increase in the fundamental prices reveals market inefficiency on the short side.

4. Consideration of the Mechanism

In this section, we discuss the mechanisms that cause inefficiencies (overvaluation) when execution prices exceed fundamental prices. In addition to $\log(M_{ie,ov}/M_{ie,uv})$, this study calculates the following three indices to examine the mechanism.

First, we calculated the board thickness (depth). More specifically, we aggregated the (unexecuted) purchase and sell orders that were within 0.6% above or below the fundamental price P_f^t before the order was placed at time *t*. Aggregation is performed every 100,000th time (50 aggregated in every simulation)[16], and the board thickness is defined as the mean value every 50 times.

Furthermore, we calculated the impact on the market. The zaraba method involves two types of executions: (A) when a purchase order is placed and (B) when a sell order is placed. More specifically,

$$\frac{1}{\#\text{ of execution by Purchase Order}}\sum\frac{\Delta\,\text{Best Ask}^t}{P_f^t}$$
$$\frac{1}{\#\text{ of execution by Sell Order}}\sum\frac{|\Delta\,\text{Best Bid}^t|}{P_f^t} \tag{10}$$

using $\Delta\,\text{Best Ask}^t$ as the range in which the best ask price changes (at time t) for (A), and $\Delta\,\text{Best Bid}^t$ as the range in which the best bid price changes in case of (B).

Furthermore, we calculated the spread between the best-quoted and fundamental prices. Then, after processing all orders, executions, or order cancelations at time *t*, with the best-quoted sell price as Best Ask^t and best-quoted purchase price as Best Bid^t, at every time (except board construction period), an average of

$$\frac{\text{Best Ask}^t - P_f^t}{P_f^t},\quad \frac{P_f^t - \text{Best Bid}^t}{P_f^t} \tag{11}$$

was used.

4.1. Results of each indicator when q is changed

Table 3 shows the results of the board thickness for each parameter q, separately for the purchaser(buyer)'s and seller's sides (Figure 3 is a bar chart of the same results). For $q = 100$, the thickness of the board is almost equal on the purchase and sell sides; however, as q decreases, the

[16] If aggregation was performed each time, the process of searching for all applicable orders would make the simulation extremely time-consuming.

number of purchase orders offered within 0.6% above and below the fundamental price P_f^t increases, while the number of sell orders decreases.

Table 3 Thickness of the board for each q

q	100	99	98	95	90
Purchaser's side	88,619	183,407	195,547	239,604	308,107
Seller's side	90,957	37,388	33,241	18,945	4,577

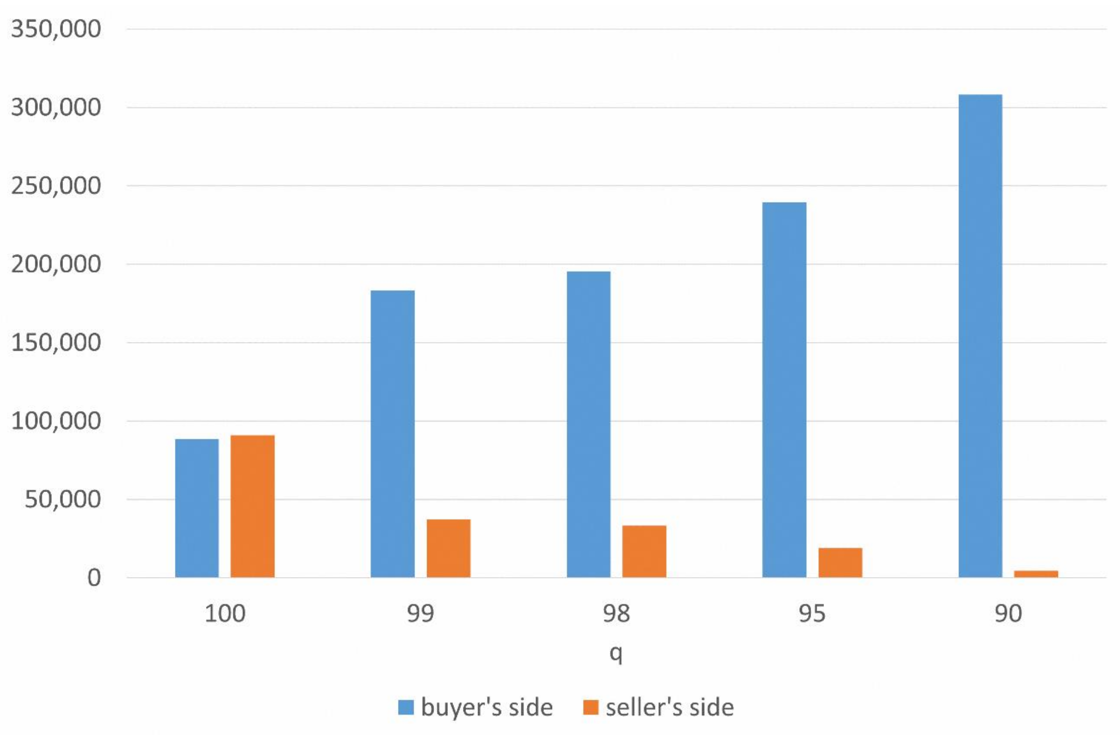

Figure 3 Thickness of board for each q

Furthermore, we show the market impact for each parameter q in Table 4 for (A) and (B) (Figure 4 shows the same results in a bar chart). If $q = 100$, the market impact is practically the same between cases (A) and (B); however, as q decreases, the rate of change in the best offer rate by the purchase order execution increases, while the rate of change in the best bid rate by the sell order execution decreases.

Furthermore, Table 5 shows the spread between each best quote and the fundamental price for each parameter q. (Figure 5 shows the same results in a line graph). As q decreases, the best offer price becomes higher than the fundamental price; however, the best bid price becomes less than the fundamental price.

Table 4 Market Impact for Each q

q	100	99	98	95	90
In case of (A)	0.0232%	0.0239%	0.0242%	0.0249%	0.0265%
In case of (B)	0.0232%	0.0204%	0.0203%	0.0199%	0.0193%

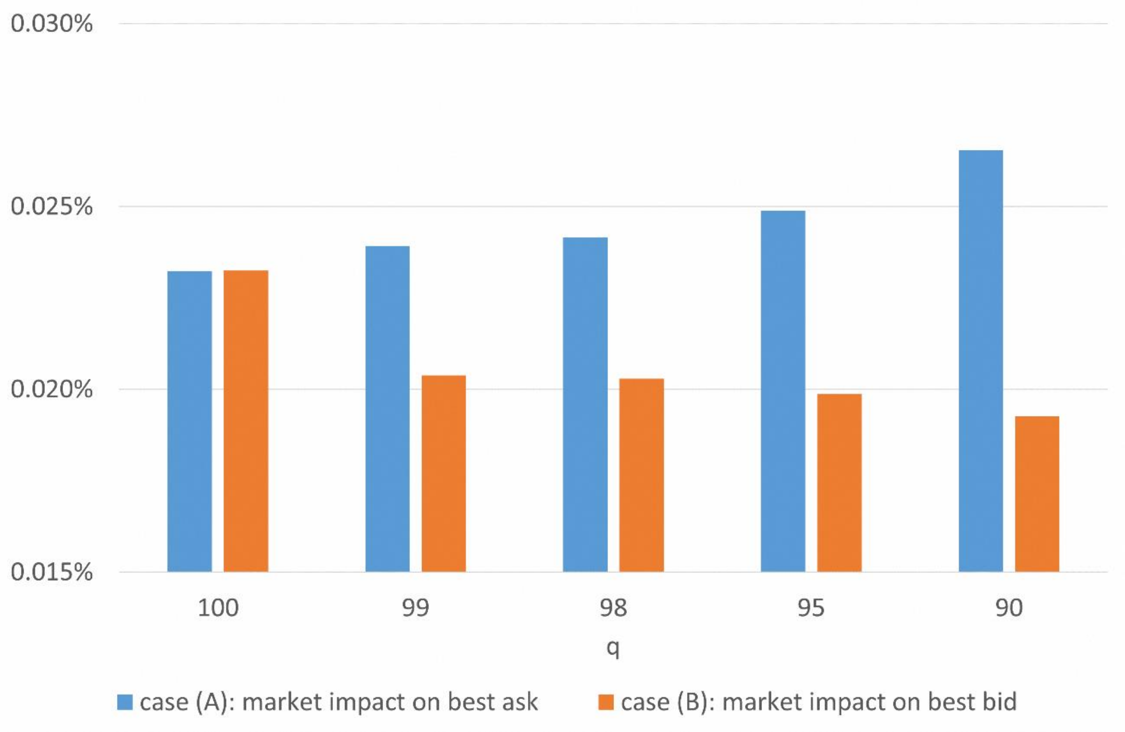

Figure 4 Market Impact of Each q

Table 5 Spread in Fundamental Price for Each q

q	100	99	98	95	90
$\frac{\text{Best Ask}^t - P_f^t}{P_f^t}$	0.067%	0.315%	0.348%	0.466%	0.694%
$\frac{P_f^t - \text{Best Bid}^t}{P_f^t}$	0.075%	−0.198%	−0.231%	−0.348%	−0.574%

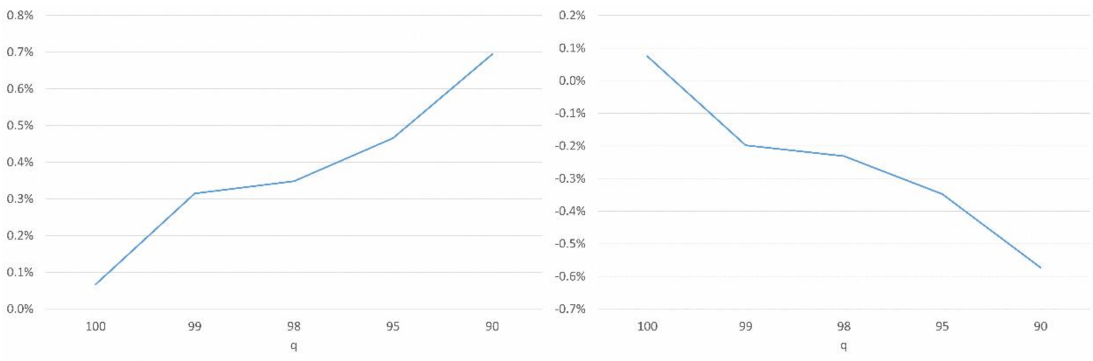

Figure 5 $\frac{\text{Best Ask}^t - P_f^t}{P_f^t}$ **(left) and** $\frac{P_f^t - \text{Best Bid}^t}{P_f^t}$ **(right) for each** q

4.2 Results of Each Indicator when Changing d

Similarly, the results of separately aggregating the board thickness for each parameter d for the purchase and sell sides (Table 6 and Figure 6), the market impact on cases (A) and (B) (Table 7 and Figure 7), and the spread between each best quote and the fundamental price (Table 8 and Figure 8) are also shown. If d increases, the effect on the results of each indicator is less significant than when $d = \pm 0$ (although small, as d increases, the number of offer orders placed in the 0.6% range above and below the fundamental price P_f^t increases, and the rate of change in the best bid price due to the execution by sell orders increases, with the spread between the best bid and fundamental price widening).

Table 6 Board thickness for each d

d	± 0	+1	+2	+5	+10
Purchaser's side	88,619	90,165	86,974	86,359	82,496
Seller's side	90,957	92,712	92,194	94,525	98,234

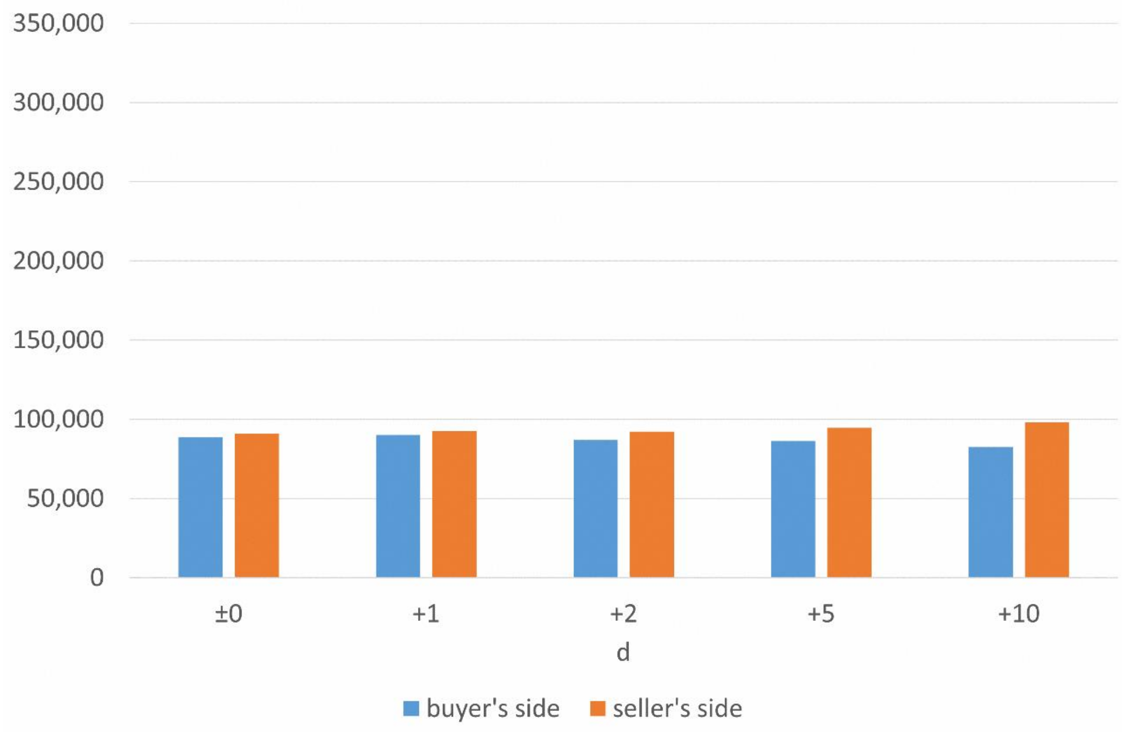

Figure 6 Board thickness for each d

Table 7 Market Impact of Each d

d	± 0	+1	+2	+5	+10
In case of (A)	0.0232%	0.0232%	0.0232%	0.0230%	0.0228%
In case of (B)	0.0232%	0.0233%	0.0234%	0.0235%	0.0237%

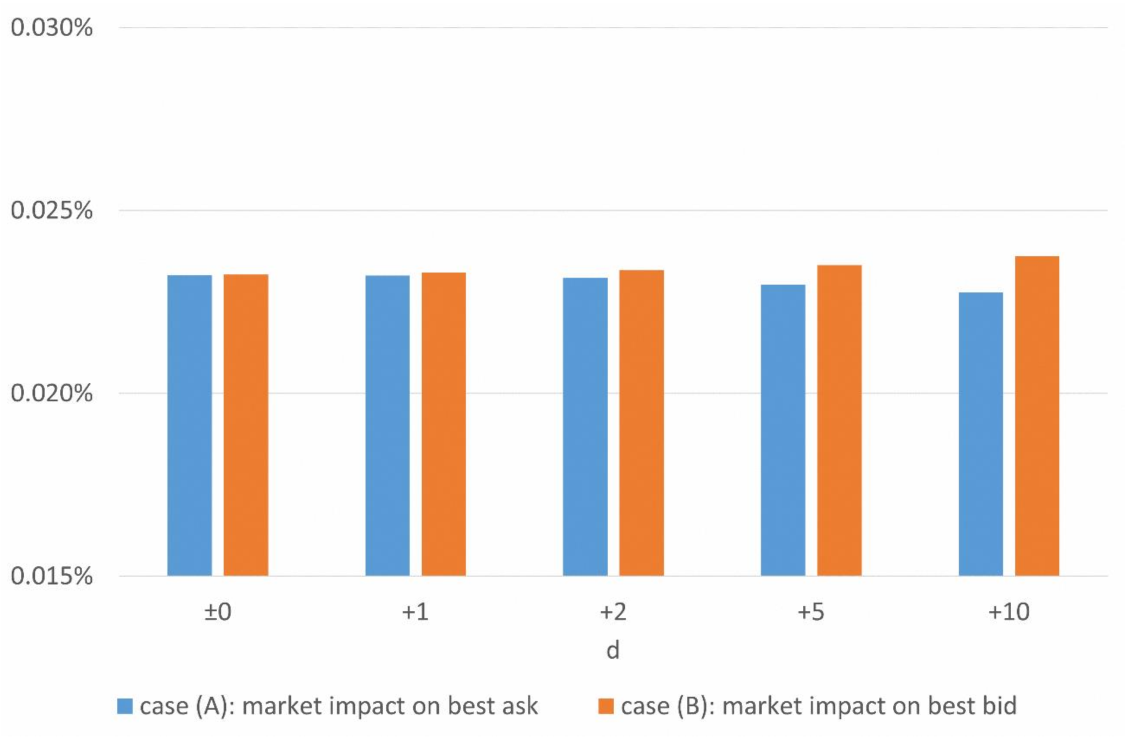

Figure 7 Market Impact of Each d

Table 8 Spread with the Fundamental Price for Each d

d	± 0	+1	+2	+5	+10
$\frac{\text{Best Ask}^t - P_f^t}{P_f^t}$	0.067%	0.066%	0.065%	0.062%	0.061%
$\frac{P_f^t - \text{Best Bid}^t}{P_f^t}$	0.075%	0.076%	0.076%	0.079%	0.081%

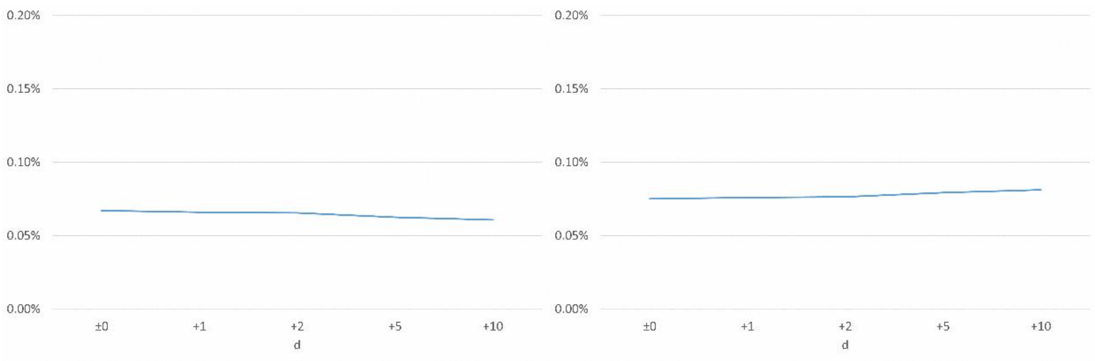

Figure 8 $\frac{\text{Best Ask}^t - P_f^t}{P_f^t}$ **(left) and** $\frac{P_f^t - \text{Best Bid}^t}{P_f^t}$ **(right) for each** $\boldsymbol{d}$

4.3. Short-Side Market Inefficiencies' Mechanisms

As q decreases from 100, $\log(M_{ie,ov}/M_{ie,uv})$ increases, the number of sell orders presented within a certain range from P_f^t decreases, the market impact on the execution of the purchase orders increases, and the best offer price become higher than the fundamental price. However, when parameter d was increased relative to the case with $d = \pm 0$, $\log(M_{ie,ov}/M_{ie,uv})$ decreased slightly, and no significant effect occurred on the difference between the purchase and sell sides of the board thickness and the market impact on (A) and (B), as well as the spread between each best quote and fundamental price (a slight reverse trend was observed compared to the results with a small q).

Therefore, we can conclude that the three indicators proposed in this section are closely related to the mechanism-presenting market inefficiency on the short side, which is consistent with the previously described results. The process by which the breakdown of the short side in market inefficiency in the artificial market model in this study became larger is as follows: the number of sell orders that remain within a certain range of the fundamental price P_f^t is reduced by reducing the parameter q, and when the agent presents a purchase order (the best ask Best Ask^{t-1} or above at time t), the change in the best ask $\Delta\,\text{Best Ask}^t$ is likely to become larger. Therefore, the best ask price Best Ask^t deviated from the fundamental price significantly, and it became easier to execute at a level higher than the fundamental price P_f^{t+1} after time $t+1$ (alternatively, the process of slightly increasing the breakdown of the long side by increasing the value of parameter d is considered to set a series of triggers that cause the fundamental price to increase even when the execution price is not updated immediately before, and the number of purchase orders within a certain range to decrease).

5. Empirical Analysis

In this section, we use order data from the TSE to verify our results. Note that there are different tendencies regarding the use of short selling in different market segments. For example, [8], using TSE execution data from 2015 to 2019, demonstrated that while the ratio of short-selling in the First Section of the market was ~30%, it was only ~5% for a group of stocks in other market segments. Based on the results of [9], [8] asserted, among other things, that the difference in the share percentage held by institutional investors (higher in the First Section of the market) affected the supply of shares in the lending market. However, the topics of how short-selling is used and the purchasing and selling ratio in the ordering process have yet to be extensively discussed. In this study, we attempt to estimate the parameter q for the First Section and other market segments.

5.1. Data and Tabulation Method

We first limited the target analysis period to all business days from January to December 2020. Then, we extracted all new and changed orders received by the TSE during the period under consideration[17]. We analyzed domestic stocks listed on the TSE. Further, we included stocks for which both sell and purchase orders were placed on all business days but excluded stocks that were listed, delisted, or changed in the market segment during the period under analysis.

We then calculated the sell/purchase ratio for each stock on each business day (Equation (12)). Then, we calculated the median value for each issue for the period under analysis to remove the effect of the results for the business days when the ratio of sell to purchase orders is extremely unbalanced.

$$\text{Sell/purchase ratio} = \frac{\text{Quantity of sell orders issued}}{\text{Quantity of purchase orders issued}} \tag{12}$$

5.2. Results

Table 9 and Figure 9 show the distribution of the median sell/purchase ratio for the analysis period, divided into the First Section and other[18] market segments[19]. The line in the center of the box indicates the median (Figure 9), and the lines above and below the box indicate the 75th and 25th percentile points, respectively. Moreover, the upper and lower whiskers indicate the 75th percentile

[17] In other words, includes orders received during unattended times.
[18] Second Section, Mothers, JASDAQ Standard, JASDAQ Growth.
[19] Second Section, Mothers, JASDAQ Standard, JASDAQ Growth.

point + 1.5 × the interquartile range and the 25th percentile point − 1.5 × the interquartile range, respectively, and stocks beyond the upper and lower whiskers are shown as outliers. The quartile range is 75%–25% points.

Table 9 Sell/purchase ratio median value statistics

	First Section of Market	Other than First Section of Market
Max value	1.496	2.086
75% points	1.018	0.973
Median	0.998	0.861
25% points	0.955	0.758
Min value	0.520	0.372
Sample size	2133	1404
>1	1011	281
<1	1119	1116
=1	3	7

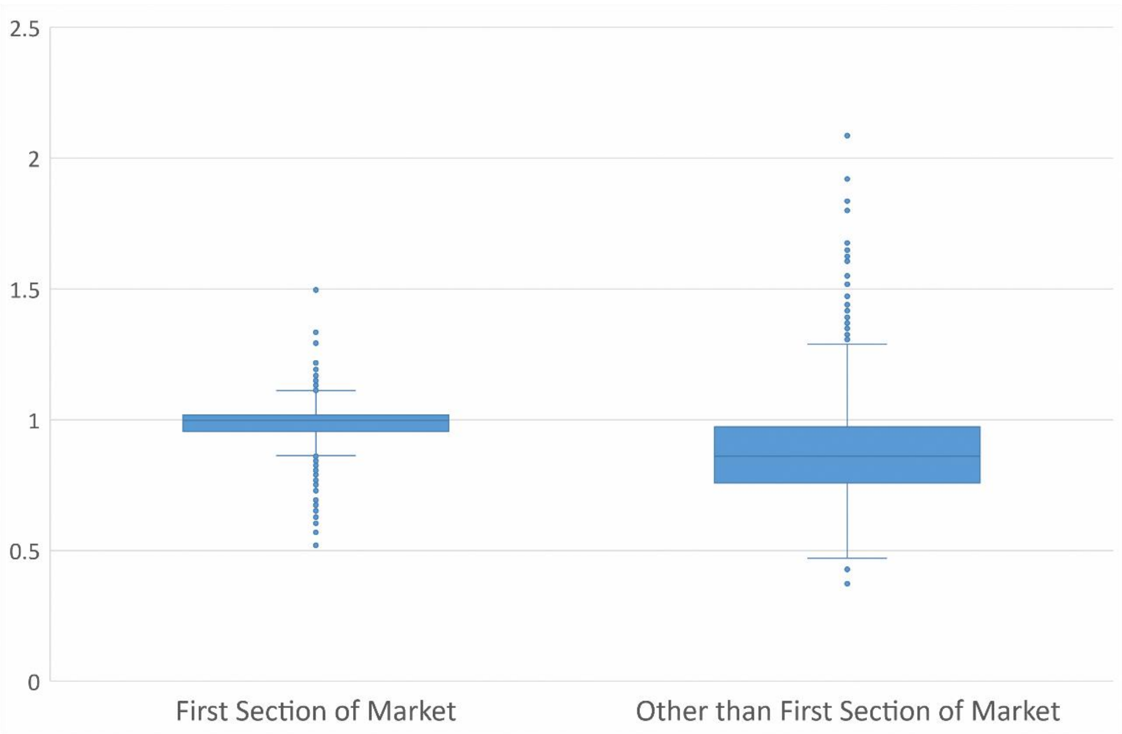

Figure 9 Sell/purchase ratio Median Value Distribution

In the First Section, the median of the sell/purchase ratio settled around one, whereas in the other market segments, the median of the sell/purchase ratio for the period under analysis was 0.861, which was less than one for ~80% of all stocks.

Considering the result validated by [8], where the ratio of short-selling was relatively high in the First Section and low in other market segments, the high ratio of short-selling could be the reason the sell/purchase ratio median is distributed around one in the First Section and the low ratio of short-selling could be the reason sell/purchase ratio median tends to be less than one in other segments.

A stock distribution survey conducted by the TSE[20] showed that the amount and percentage of stocks held by domestic financial institutions at the end of 2020 were ~220 trillion yen, or 30.7%, in the First Section, and ~two trillion yen, or 8.0%, in the other sections of the market. Based on this, it can be inferred that market segments other than the First Section were more constrained in placing orders for short selling considering the supply of stocks to the lending market by institutional investors.

The above results showed a tendency for the number of sell orders to be smaller than the number of purchase orders in terms of the total number of orders placed (except for certain stocks) in the market segments where the ratio of short-selling is considered low. When compared with the artificial market model of this study (see 2.2 above), in which a situation is simulated in which selling orders are constrained, the parameter q for stocks in the First Section can be considered close to 100 under normal conditions; however, in other market segments, the situation may have continued in which the parameter q for many stocks is less than 100.

[20] https://www.jpx.co.jp/markets/statistics-equities/examination/01.html

6. Summary

Two hypotheses are described in this study: the first hypothesis is that inefficiencies on the short side, where the execution price exceeds the fundamental price and manifest themselves because of the less investment in overvalued stocks (short-selling), and the second one is that the information released to investors is biased (includes more positive content). Therefore, we used the artificial market model reported in [6] to simulate the case where only the number of sell orders is reduced and the fundamental price is increased moderately. Simultaneously, we measured the breakdown of the short and long sides of market inefficiency and confirmed that the breakdown on the short side is larger when only the volume of sell orders is reduced (this is consistent with the results of [7]). However, when the fundamental price increase was moderate, this result could not be confirmed.

Furthermore, the mechanism by which the short side market inefficiencies identified in this study are assumed to manifest themselves is that the total number of sell orders presented within a certain range from the fundamental price (on the order board) decreases, and the market impact on purchase orders (when the order price is high) increases; therefore, the best offer price diverges from the fundamental price to a greater degree, and it is easier to execute at a relatively high price.

Additionally, our empirical analysis of the TSE revealed that, in the First Section, the median of the sell/purchase ratio in terms of order volume settled around one, whereas in other market segments, the volume of sell orders tended to be lower than that of purchase orders for several stocks.

In terms of future issues, there is a simulation of fundamental price decline patterns (including sharp drops). This study was unable to confirm the relationship between a rise in fundamental prices and a larger breakdown of market inefficiency on the short side, but short side market inefficiencies may be significant if the fundamental price drops to a level below the contract price. There would also be value in analyzing thresholds or how large the decline in fundamental prices would have to be for the short-side breakdown to be larger. In the future, an analysis of how market inefficiencies due to the short side can be eliminated should be considered. This study does not mention or discuss the process by which short side market inefficiencies can be eliminated.

References

[1] Gastineau, Gary L., Olma, Andrew R., and Zielinski, Robert G.: Equity Portfolio Management, in *CFA Program Curriculum: level III, 2018*, Vol.4, pp.254-324 (2017)

[2] Cohen, Lauren., Malloy, Christopher., and Nguyen, Quoc.,: Lazy Prices, *The Journal of Finance*, Vol.75, No.3, pp.1371-1415 (2020)

[3] Womack, Kent L.,: Do Brokerage Analysts' Recommendations Have Investment Value?, *The Journal of Finance*, Vol.51, No.1, pp.137-167 (1996)

[4] Chen, S., Chang, C., and Du, Y.,: Agent-based economic models and econometrics, *The Knowledge Engineering Review*, 27(2), pp.187-219 (2012)

[5] Cont, R.,: Empirical Properties of Asset Returns: Stylized Facts and Statistical Issues, *Quantitative Finance*, Vol.1, No.2, pp.223-236 (2001)

[6] Mizuta, T., Noritake, Y., Hayakawa, S., Izumi, K.,: Impacts of Speedup of Market System on Price Formations using Artificial Market Simulations, *JPX working paper,* Vol.9, Japan Exchange Group, 2015

[7] Mizuta, T., Izumi, K., Yagi, I., Yoshimura, S.,: Investigation of Price Variation Limits, Short Selling Regulation, and Uptick Rules and Their Optimal Design by Artificial Market Simulations, *Electronics and Communications in Japan*, Vol.98, No.7, pp.13-21, (2015)

[8] Sakurai, S., Uehara, D., Hosaka, G.,: Analysis of Detail Breakdown Trading Data (Margin Transactions and Short Selling), *JPX working paper*, Vol.35, Japan Exchange Group, 2021

[9] Uno, J., Umeno, J., Muroi, R.,: Empirical analysis of the Japanese stock lending market: Estimation of effect of tightening of short selling regulations, *Securities Analysts Journal*, June 2009

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Appendix A Results of reproducing stylized facts, etc.

The artificial market model in [6] is designed such that it can reproduce fat tail and volatility clustering as typical stylized factors. We confirmed that the reproduction results are not impaired by changing parameters q and d. The fat tail criterion indicates the kurtosis of the return to be positive[21], whereas the volatility clustering criterion indicates that the squared return autocorrelation is positive[5][22].

Table 9 Stylized fact for each q

q		100	99	98	95	90
Kurtosis		15.13	14.11	14.09	14.08	14.09
Autocorrelation	Lag1	0.052	0.047	0.046	0.047	0.047
	Lag2	0.052	0.046	0.046	0.046	0.046
	Lag3	0.050	0.044	0.043	0.044	0.044
	Lag4	0.047	0.042	0.042	0.042	0.042
	Lag5	0.044	0.040	0.040	0.040	0.040

Table 10 Stylized fact for each d

d		+1	+2	+5	+10
Kurtosis		15.12	15.13	15.12	15.13
Autocorrelation	Lag1	0.052	0.052	0.052	0.052
	Lag2	0.052	0.052	0.052	0.052
	Lag3	0.050	0.050	0.050	0.050
	Lag4	0.048	0.048	0.048	0.047
	Lag5	0.045	0.044	0.045	0.045

Tables 10 and 11 show the averages over 50 times for each statistic and pattern[23]. The individual results are omitted, but all q and d patterns reproduced fat tail and volatility clustering on each of the 50 times. For reference, Tables 12 and 13 show the immediate execution rate[24] of orders and the cancelation rate up to $t = t_e$ (by purchasing and selling in terms of volume) when the parameters q

[21] When the kurtosis of normal distribution is 0.

[22] The statistics were calculated using the returns for each elapsed time 1 (excluding the board construction period).

[23] When $d = \pm 0$, the results are the same as in the case of $q = 100$.

[24] Definition of immediate execution rate: Volume of orders executed immediately after being placed (without remaining on the board) divided by the total volume of orders placed.

and d are changed. These results were averaged over 50 times.

Table 11 Order immediate execution rate and cancelation rate for each q

q	Immediate execution rate		Cancelation rate	
	Purchase orders	Sell orders	Purchase orders	Sell orders
100	32.32%	32.31%	35.17%	35.17%
99	31.33%	33.13%	36.10%	35.03%
98	31.09%	33.37%	36.43%	34.72%
95	30.44%	34.30%	37.18%	33.47%
90	29.21%	35.94%	38.60%	31.42%

Table 12 Order immediate execution rate and cancelation rate for each d

d	Immediate execution rate		Cancelation rate	
	Purchase orders	Sell orders	Purchase orders	Sell orders
+1	32.35%	32.23%	35.23%	35.23%
+2	32.39%	32.17%	35.25%	35.23%
+5	32.55%	32.03%	35.22%	35.23%
+10	32.78%	31.80%	35.21%	35.22%

Appendix B Asymmetry of market inefficiencies over time series

The $\log(M_{ie,ov}/M_{ie,uv})$ shown in the main text is the result for time $t = t_e$, and the results of measuring the asymmetry of market inefficiency using the same calculation method for every 100,000 times is shown in Figure 10. In this study, note that the breakdown on the short side was larger when parameter q was reduced, so the results are presented for the pattern in Equation (5). When $q < 100$, the asymmetry value increases until time 300,000, but after that, the pace of increase slows down. Furthermore, there was no indication that the market inefficiency on the short side was eliminated with time. Based on this, we can surmise that the process by which market inefficiency occurs on the short side and the process by which market inefficiency is eliminated are two separate processes.

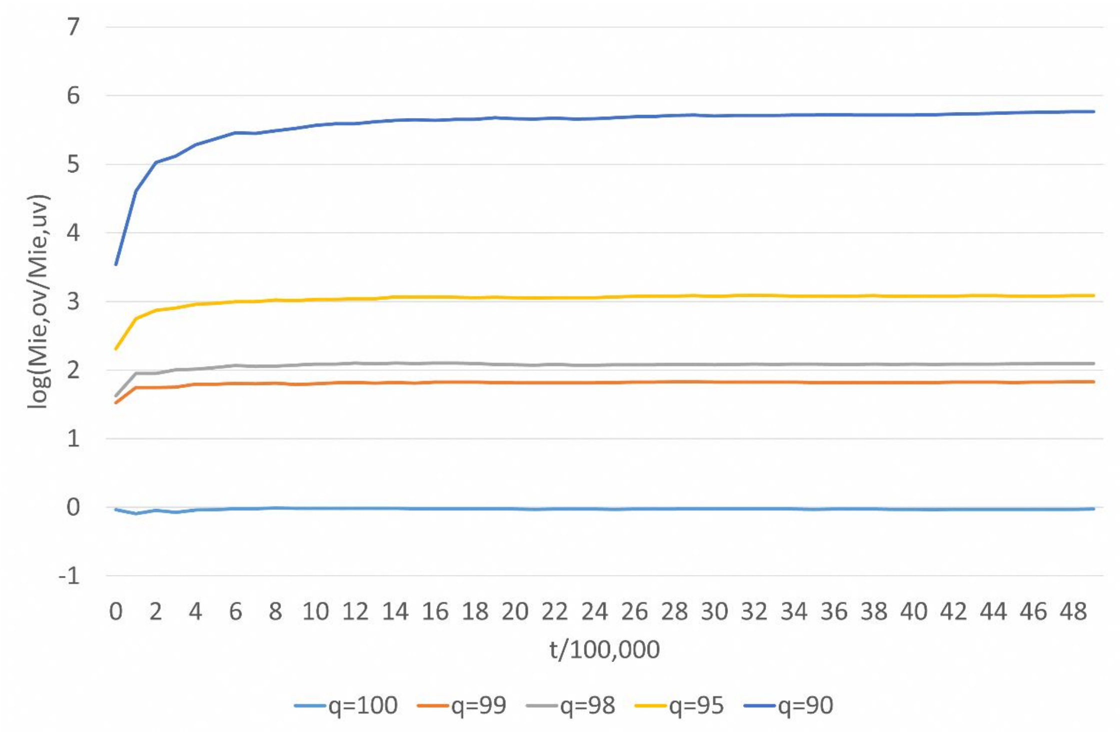

Figure 10 $\log(M_{ie,ov}/M_{ie,uv})$ for each 100,000 times

Appendix C Monthly transition in the sell/purchase ratio for orders placed

We conducted the same empirical analysis for each month of the period under analysis when calculating the sell/purchase ratio shown in Equation (12). The results for the First Section of the market are shown in Table 14 and Figure 11, whereas the results for other market categories are shown in Table 15 and Figure 12.

For every month, the median sell/purchase ratio settled around one for the First Section and less than one for the other market categories. Additionally, although there is monthly variation, many stocks in the sample (sample size), with the exception of the First Section, have a median sell/purchase ratio (within the analysis period) below one.

Table 13 Sell/Purchase Ratio Median Statistics (First Section of Market, monthly)

	Jan	Feb	March	April	May	June	July	Aug	Sept	Oct	Nov	Dec
Max value	1.753	1.890	1.573	1.776	1.895	2.013	1.760	1.866	2.147	3.241	2.053	1.681
75% points	1.005	1.011	1.048	1.029	1.032	1.050	1.063	1.036	1.021	1.066	1.038	1.035
Median	0.954	0.965	1.009	0.993	0.992	1.010	1.019	0.991	0.978	1.019	0.998	0.997
25% points	0.878	0.887	0.953	0.949	0.937	0.961	0.968	0.929	0.915	0.969	0.941	0.945
Min value	0.324	0.233	0.413	0.272	0.111	0.266	0.382	0.278	0.237	0.189	0.165	0.147
Sample size	2158	2158	2159	2166	2169	2166	2167	2171	2170	2176	2174	2174
>1	584	672	1207	969	979	1224	1318	975	817	1355	1066	1037
<1	1573	1486	952	1194	1190	942	846	1196	1353	821	1107	1137
=1	1	0	0	3	0	0	3	0	0	0	1	0

Table 14 Sell/Purchase Ratio Median Statistics (Other market categories, monthly)

	Jan	Feb	March	April	May	June	July	Aug	Sept	Oct	Nov	Dec
Max value	3.286	3.078	5.172	4.961	4.199	4.584	3.705	3.745	3.444	3.865	3.600	4.198
75% points	0.941	0.919	1.027	0.993	1.022	1.083	1.116	1.042	1.055	1.081	1.062	1.035
Median	0.771	0.730	0.853	0.846	0.874	0.919	0.929	0.855	0.859	0.877	0.862	0.854
25% points	0.638	0.572	0.716	0.710	0.734	0.768	0.765	0.710	0.693	0.695	0.681	0.675
Min value	0.109	0.075	0.054	0.055	0.089	0.070	0.244	0.142	0.080	0.164	0.125	0.135
Sample size	1506	1499	1471	1493	1500	1493	1493	1499	1495	1500	1506	1499
>1	282	281	403	350	414	538	569	444	444	509	484	431
<1	1222	1218	1060	1135	1085	954	915	1053	1047	988	1017	1068
=1	2	0	8	8	1	1	9	2	4	3	5	0

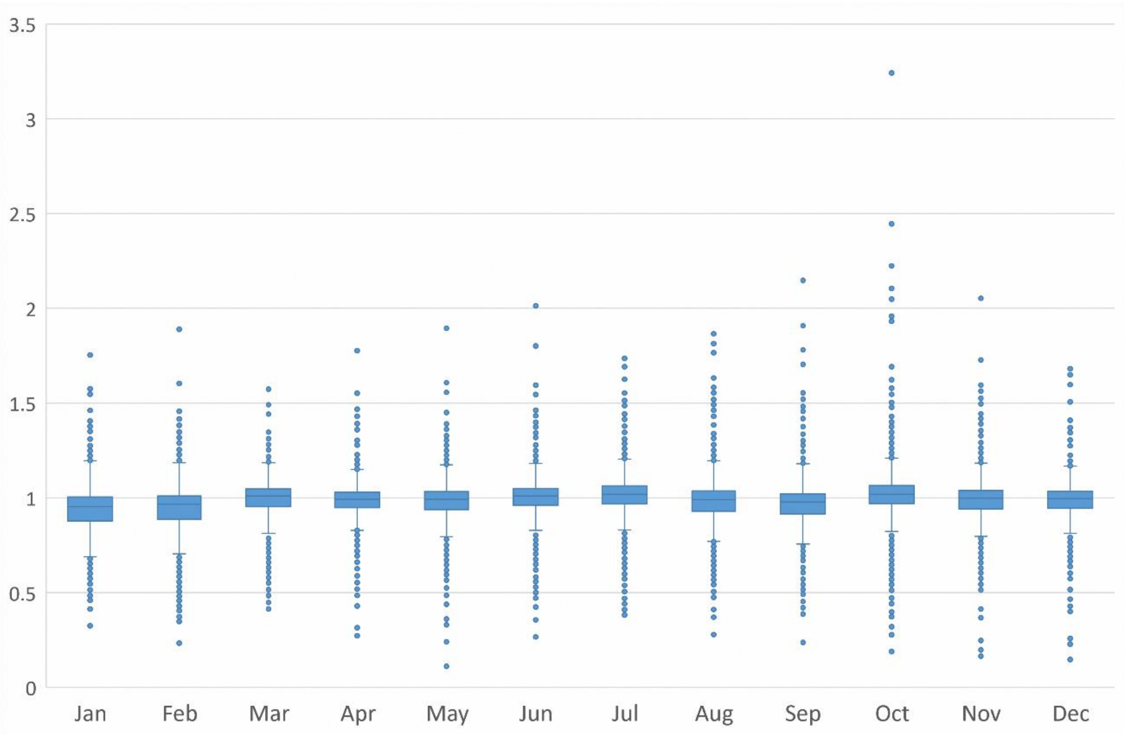

Figure 11 Sell/Purchase ratio, Median value distribution (First Section of Market, monthly)

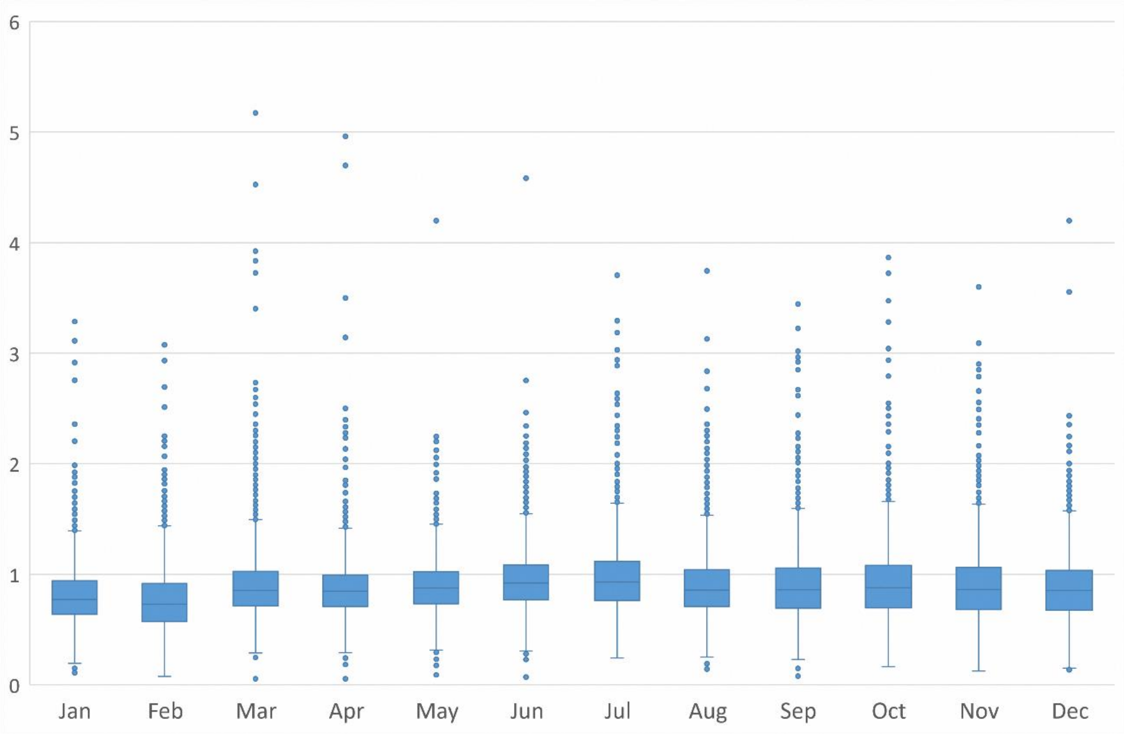

Figure 12 Sell/Purchase ratio, Median value distribution (Other market categories, monthly)