Analysis of Short Side Market Inefficiencies Using Artificial Markets (abridged version)

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Overview

- ➢ Focus on market inefficiencies when simulating artificial markets

  - ◗ Market inefficiency： （in a simulation） Difference between executed price and fundamental price

- ➢ Factors that cause the breakdown of the short-side divergence and to manifest as being overvalued, i.e., contract price > fundamental price.

  - ◗ In terms of short selling constraints, we performed empirical analysis of the ratio of sell/ purchase orders using real stock market order data.

Factors that reveal market inefficiencies on the (overvalued) short side (1/2)

1. Restrictions on (stock) short selling
   - Sell orders
     - Restrictions on placing orders, especially for short sales (insufficient stock via lending market and procurement costs)
     - Investors may anticipate a drop in the stock price but do not actually place the order as a short sale
   - Purchase orders
     - If you have cash, there are no restrictions on orders

➤ Could it be the case that the total number of orders to sell is less than the number of orders to purchase?

Factors that reveal market inefficiencies on the (overvalued) short side (2/2)

2. Trends in published information

   - Equity portfolios that were shorting the stock of group companies that had changed their accounting reporting terminology and structure and purchasing the stock of group companies that had not changed earned returns.

     - Cohen et al.(2020)

   - In the brokerage firm issued reports, there were more purchase recommendations than new sell recommendations.

     - Womack(1996)

➢ There are exceptional situations where a lot of information related to shorting is published, but does that really have an impact on the stock evaluated by investors?

What kind of simulation is performed?

1. Restrictions on short selling

   - Reducing the volume of sell orders placed by an agent in a transaction to a quantity less than purchase orders = no restrictions on purchase orders, only sell orders.

2. Bias in published information

   - Gradual increase in the fundamental price used by the agent when determining the order price

➢ Validate whether changes to the sell order quantity and fundamental price, respectively, result in large overvalued deviations (execution price > fundamental price), and look at the mechanism of such deviations.

Thinking behind the construction of an artificial market model

- We constructed a model that was as simple as possible, within the scope of a model that achieved the following objectives.

    - Know what the impact would be if 1. Sell order quantity or 2. Fundamental price increase degree were individually changed.

    - If there is any impact, investigate what kind of mechanism this would be

- The purpose is not to completely reproduce the actual market

- Assuming normal market transactions, determine the parameters for 1. and 2. so that there would be no significant difference from the original values.

    - If extreme parameters are used in the simulation, this may result in frequent and abrupt price fluctuations, and the corresponding results may differ from those of the real stock market, where there is a mix of different execution methods.

Agent

- Predicted return

$$r_{e,j}^t = \frac{1}{w_{1,j}+w_{2,j}+u_j}\left(w_{1,j}\log\frac{P_f^t}{P^{t-1}} + w_{2,j}\log\frac{P^{t-1}}{P^{t-\tau_j-1}} + u_j\epsilon_j^t\right)$$

- Predicted price

$$P_{e,j}^t = P^{t-1}\exp(r_{e,j}^t)$$

- Order price

$$P_{o,j}^t \sim N\left(P_{e,j}^t, P_\delta^t\right)$$
$$P_\delta^t = P_{e,j}^t \times \sigma$$

- Separate selling/purchasing

$$P_{o,j}^t > P_{o,j}^t \Rightarrow \text{Buy } 100$$
$$P_{o,j}^t < P_{o,j}^t \Rightarrow \text{Sell } q$$

- ➤ Prepare 1000 of the above agents

Constructed with reference to the artificial market model of Mizuta (2015) et al.

Transaction Overview

- ➢ Transaction targets
  - ➡ 1 security with a given fundamental price at each time (order submission time)
  - ➡ Presumes no change in price due to corporate action
- ➢ Matching method
  - ➡ Continuous double auction (zaraba) method
  - ➡ Defines price executed by order at time t as P^t
- ➢ Tick size

$$\delta P = 10$$

- ➢ Number of times order made
  - ➡ 5,000 times per agent
  - ➡ Simulation end time $t_e = 5{,}000 \times 1{,}000$
  - ➡ Each agent can hold up to 20 orders (any left after that are cancelled) $\rightarrow board\ construction\ period = 20 \times 1{,}000$

Parameters

- Quantity of sell orders（quantity of purchase orders fixed at 100（unit））
$$q = \{100, 99, 98, 95, 90\}$$

- Rate of increase of fundamental price
$$\begin{cases} P_f^t = 10{,}000{,}000 \quad (t \leq board\ construction\ period) \\ P_f^t = 10{,}000{,}000 \left(1 + d\,\% \times \dfrac{t - board\ construction\ period}{t_e - board\ construction\ period}\right) \quad (t > board\ construction\ period) \end{cases}$$
$$d = \{\pm 0, +1, +2, +5, +10\}$$

- Simulations were performed 50 times for each of the nine conditions by changing the random variable table.
  - ① Case in which only the sell order quantity (q) is decreased
  - ② Case in which only the fundamental price increase rate (d) is raised
  - ③ Case in which no change was made

Evaluating Results

➥ Asymmetry of market inefficiencies

$$M_{ie,ov} = \sum_{\substack{1 \leq t \leq t_e \\ P^t > P_f^t}} \frac{|P^t - P_f^t|}{P_f^t}$$

$$M_{ie,uv} = \sum_{\substack{1 \leq t \leq t_e \\ P^t < P_f^t}} \frac{|P^t - P_f^t|}{P_f^t}$$

The larger log $\left(M_{ie,OV}/M_{ie,UV} \right)$ is greater than 1, the more this indicates a larger breakdown on the short side of market inefficiency.

Evaluating Results（Other Indicators）

➢ Thickness (depth) of board before order

- Total volume of unexecuted sell and purchase orders provided within 0.6% above and below P_f^t

- Aggregated whenever time passes 100,000 and results are averaged for each period separately for purchasing and selling ＝board thickness

Evaluating Results（Other Indicators②）

- Percentage change in best quote price as a result of execution (market impact)

  The following are averaged for all trades every time (A) a purchase order is executed and (B) a sell order is executed

  $$(A)\ \frac{\Delta \text{Best Ask}^t}{P_f^t} \qquad (B)\ \frac{|\Delta \text{Best Bid}^t|}{P_f^t}$$

- Spread of best quote price and fundamental price after order

  The following values are averaged for each time respectively after the board construction period.

  $$\frac{\text{Best Ask}^t - P_f^t}{P_f^t} \qquad \frac{P_f^t - \text{Best Bid}^t}{P_f^t}$$
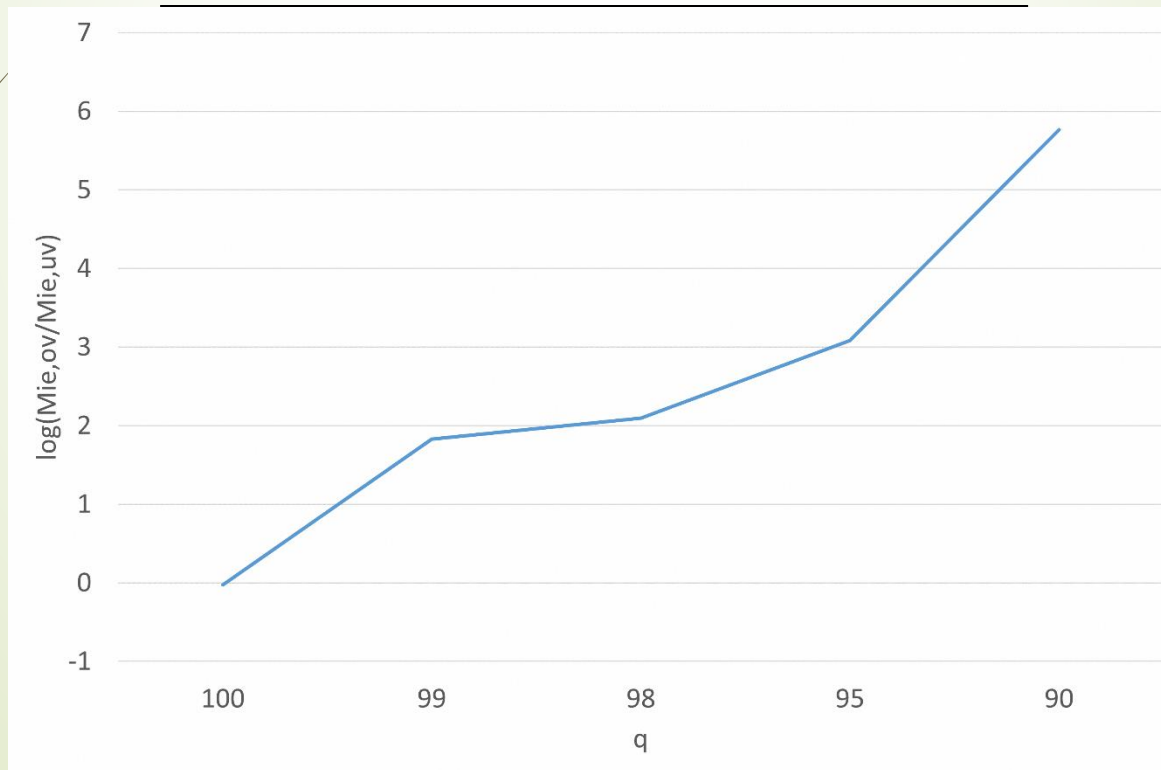
①Summary of cases in which only quantity of sell orders was decreased

Impact of restrictions on short selling

$\log\left(M_{ie,OV}/M_{ie,UV}\right)$ mean value

➢ The smaller the sell order volume, the greater the breakdown will be on the short side of market inefficiency.
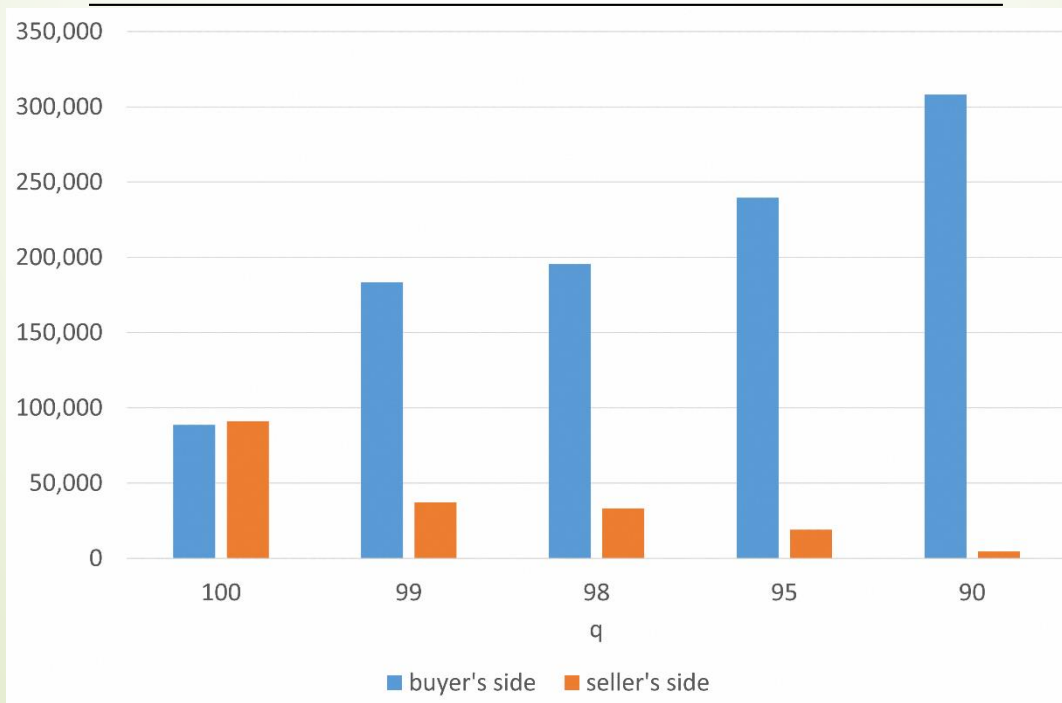
q	100	99	98	95	90
Mean Value	-0.027	1.827	2.095	3.082	5.765

Mean value of board thickness

➢ The smaller parameter q is the larger the volume of orders presented on the purchase (buyer) side (within fixed scope from fundamental value P_f^t) .

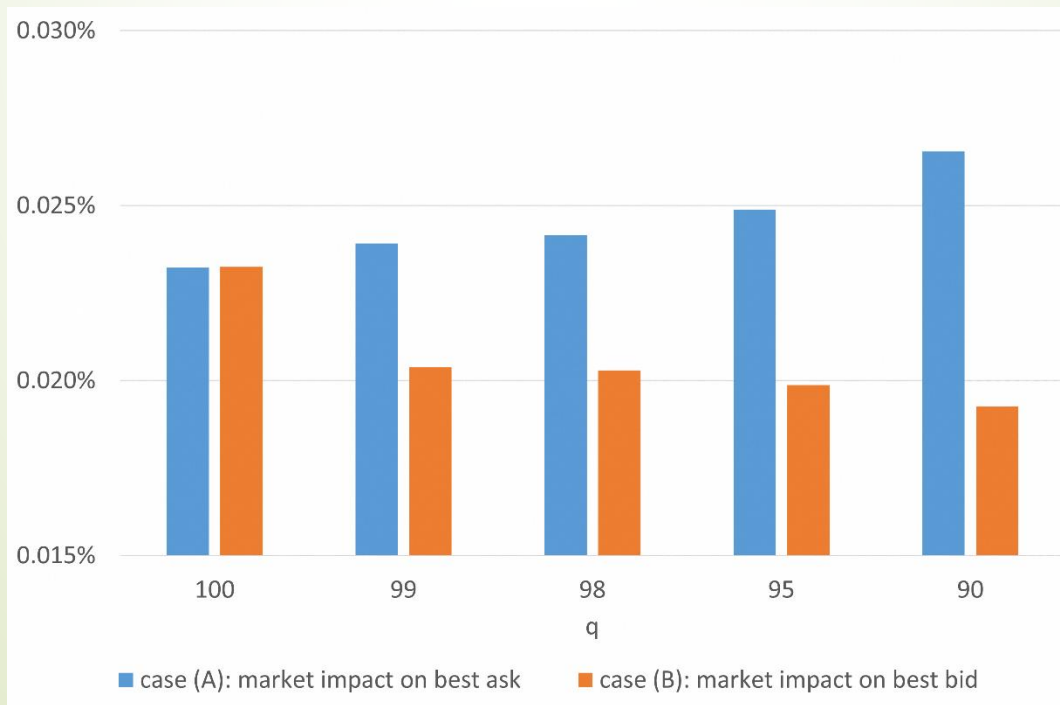| q	100	99	98	95	90
buyer's side	88,619	183,407	195,547	239,604	308,107
seller's side	90,957	37,388	33,241	18,945	4,577

Mean percentage change in price of best quote as a result of execution

➢ The smaller parameter q, the greater the market impact of the purchase order, that is to say the change in the best quoted price (increase)

q	100	99	98	95	90
Case of (A)	0.0232%	0.0239%	0.0242%	0.0249%	0.0265%
Case of (B)	0.0232%	0.0204%	0.0203%	0.0199%	0.0193%

Note : absolute values

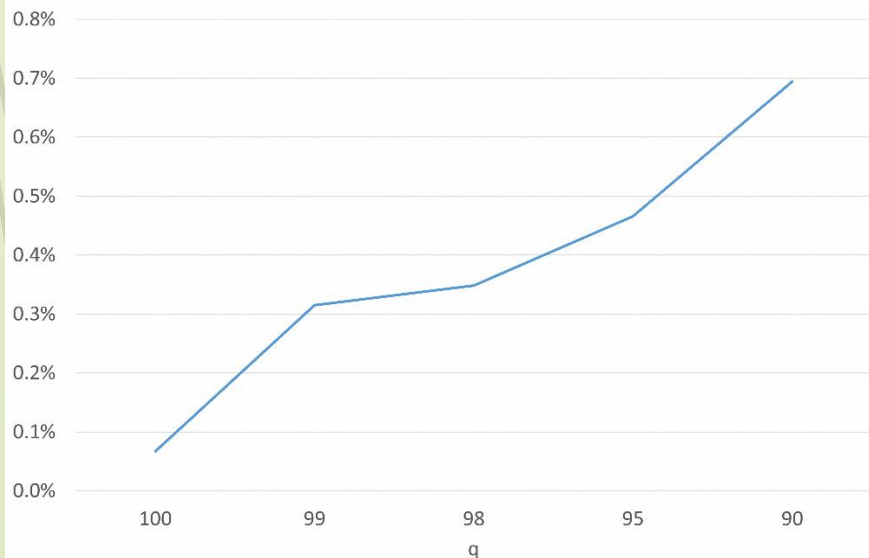case (A): market impact on best ask   case (B): market impact on best bid

Mean of spread between the best quote price and the fundamental price

➢ The smaller the parameter q, (after order), the more the best quoted price deviates higher than the fundamental price.

$$\frac{\text{Best Ask}^t - P_f^t}{P_f^t}$$

q	100	99	98	95	90
Mean Value	0.067%	0.315%	0.348%	0.466%	0.694%

$$\frac{P_f^t - \text{Best Bid}^t}{P_f^t}$$

q	100	99	98	95	90
Mean Value	0.075%	-0.198%	-0.231%	-0.348%	-0.574%
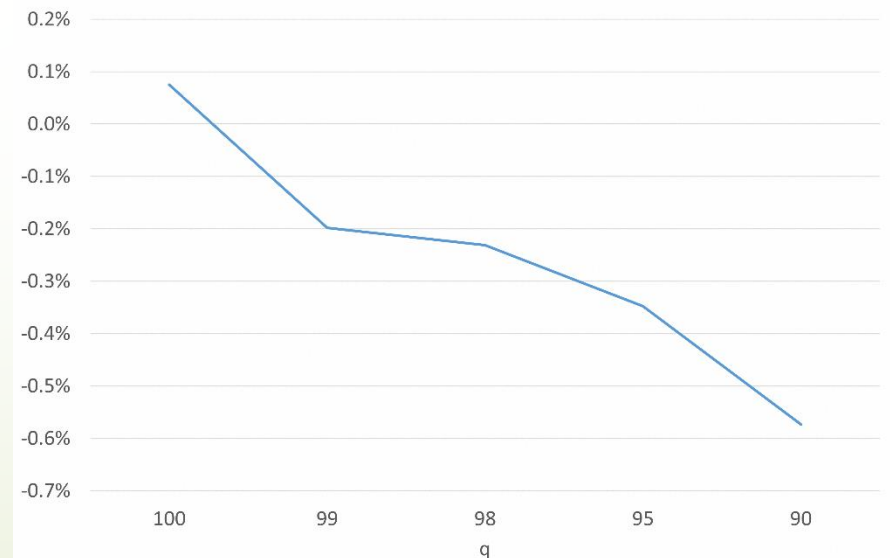
Mechanisms by which short-side market efficiencies present themselves

- The mechanism by which the short-side breakdown of market inefficiency becomes larger when the volume of sell orders is small is thought to be as follows.

  1. As the sell order volume placed is less than the purchase order volume, the volume of orders presented to the sell side is also less when comparing board thickness

  2. The change in the best offer price is greater when a (A) purchase order is placed and the execution takes place.

  3. There is a high divergence from best offer price from the fundamental price => more likely that subsequent execution will occur at a price higher than the fundamental price.

② Aggregate of cases that only have higher fundamental price increases

Impact of trends in published information

$\log\left({}^{M_{ie,OV}}\big/{}_{M_{ie,UV}}\right)$ Mean Value

➤ No results with a higher rate of increase in fundamental prices, which would have resulted in a larger breakdown of the short side of market inefficiencies were identified

d	± 0	+ 1	+ 2	+ 5	+ 10
Mean Value	-0.027	-0.033	-0.036	-0.054	-0.063
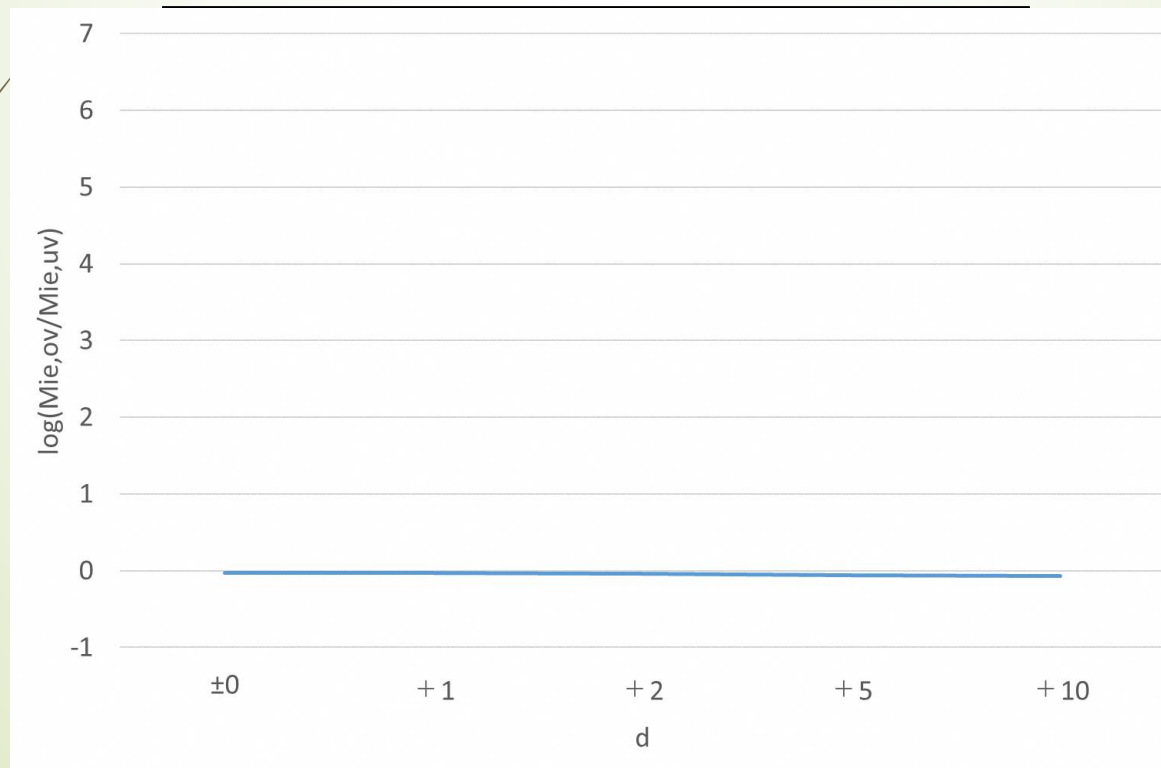
Board thickness mean value

➤ Unlike the case when parameter q is decreased, there is no significant difference in quantity on sell side and purchase (buyer) side.

d	± 0	+1	+2	+5	+10
buyer's side	88,619	90,165	86,974	86,359	82,496
seller's side	90,957	92,712	92,194	94,525	98,234
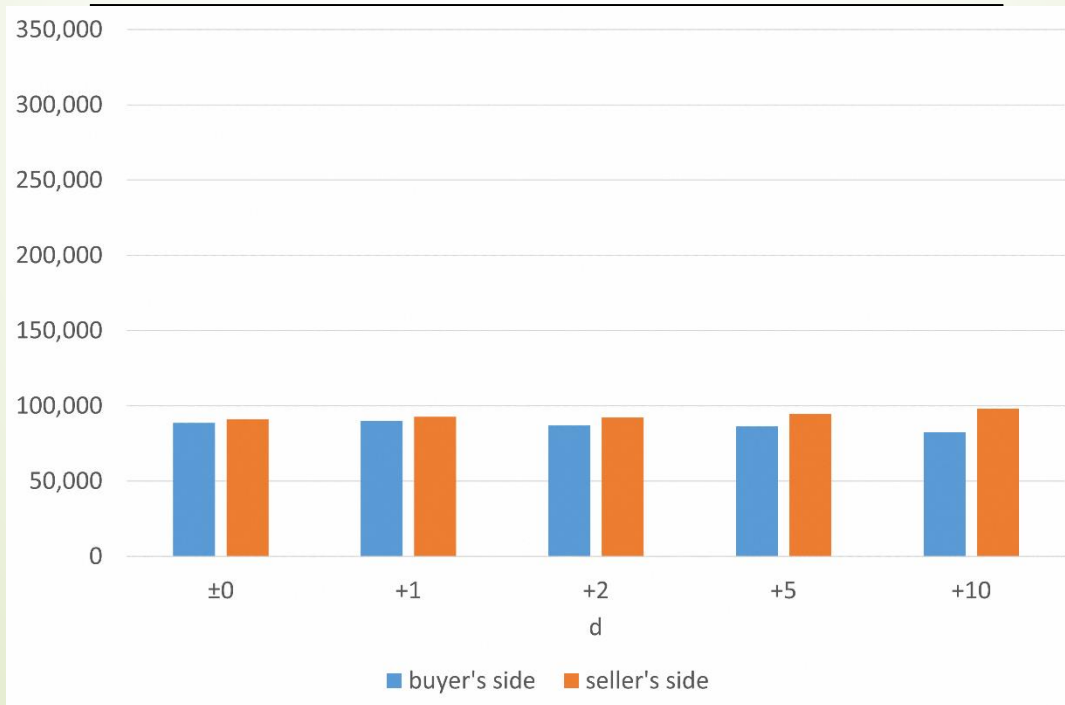
Average value of percentage change in price of best quote as a result of execution

➢ A higher parameter d does not lead to a greater market impact in the purchase order, that is to say, cause an increase in the best offer price.

d	±0	+1	+2	+5	+10
Case of (A)	0.0232%	0.0232%	0.0232%	0.0230%	0.0228%
Case of (B)	0.0232%	0.0233%	0.0234%	0.0235%	0.0237%

Note : absolute values

case (A): market impact on best ask   case (B): market impact on best bid
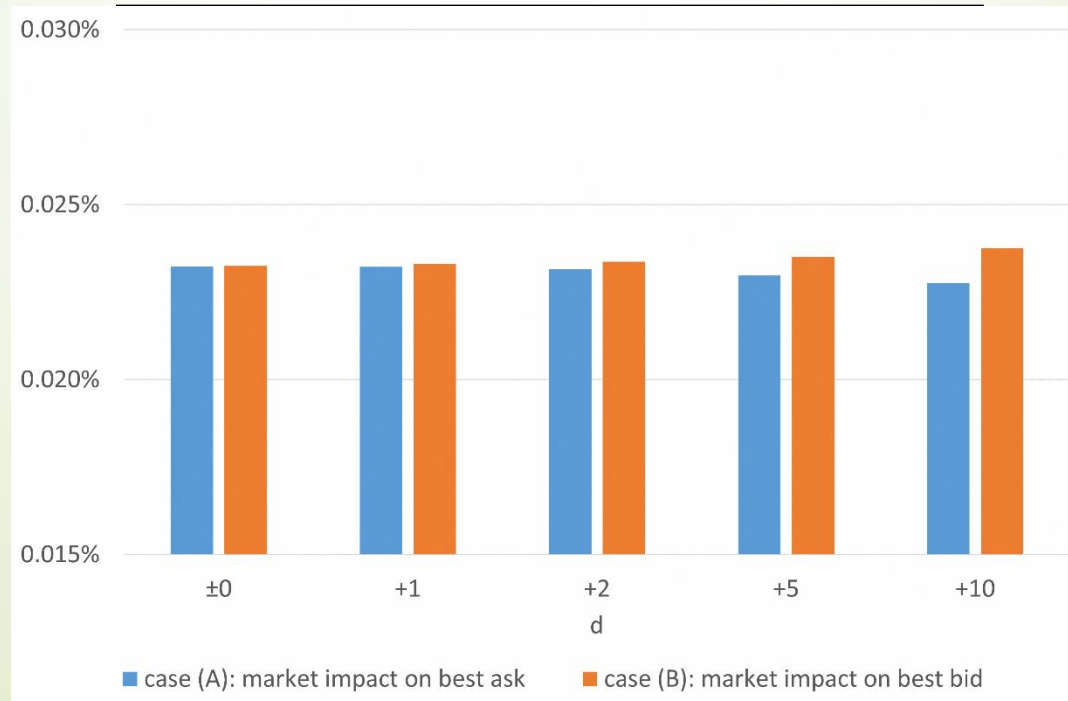
Mean value of spread between best quoted price and fundamental price

➢ Even if parameter *d* is increased, the price of the best quoted price does not deviate higher compared to the fundamental price.

$$\frac{\text{Best Ask}^t - P_f^t}{P_f^t}$$

d	± 0	+1	+2	+5	+10
Mean Value	0.067%	0.066%	0.065%	0.062%	0.061%
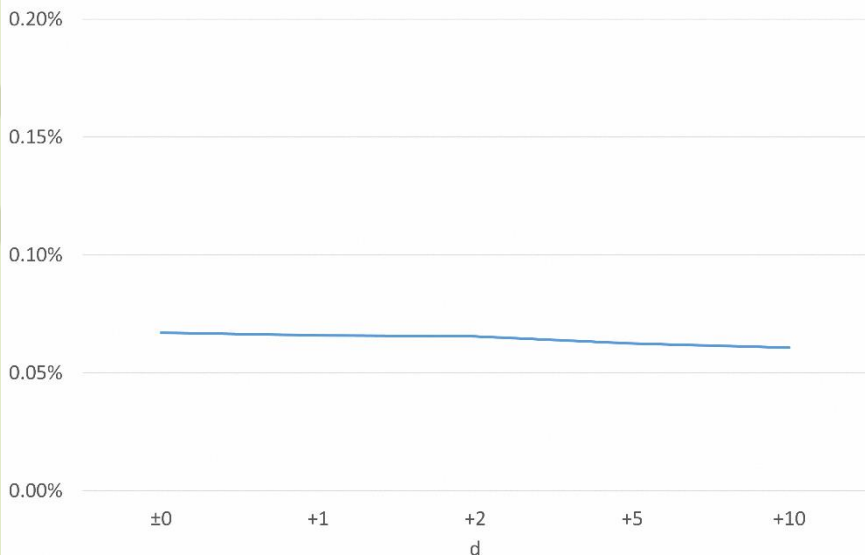
$$\frac{P_f^t - \text{Best Bid}^t}{P_f^t}$$

d	± 0	+1	+2	+5	+10
Mean Value	0.075%	0.076%	0.076%	0.079%	0.081%
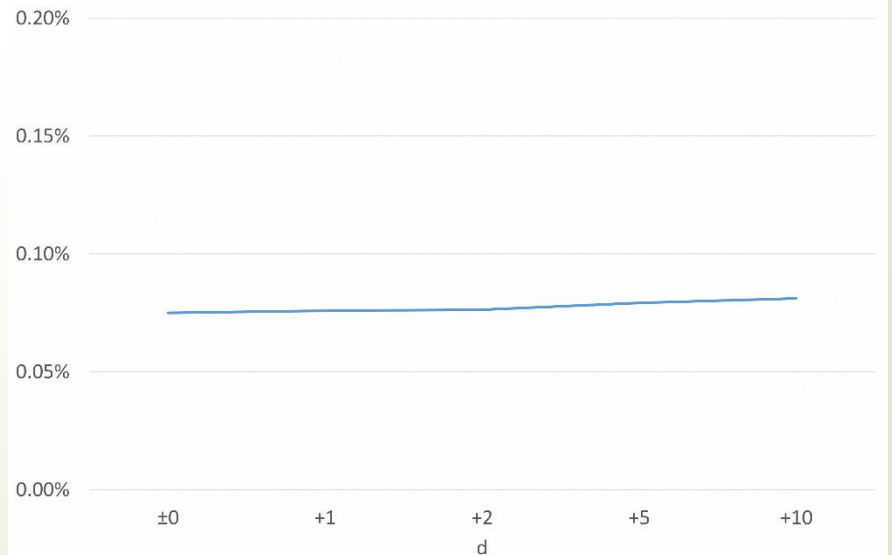
③ Estimation of sell/purchase order ratio in the actual stock market (by market segment)

Empirical analysis using actual Tokyo Stock Exchange order data

Overview of Empirical Analysis

➢ We calculated the sell/purchase ratio for domestic stocks listed on the Tokyo Stock Exchange for each business day from January to December 2020, as well as the median for each issue.

- Sell/purchase Ratio = Volume of Sell Orders Placed / Volume of purchase Orders Placed

- Covers all new and modified orders

- This also covers stocks for which both sell and purchase orders can be placed on any business day

➢ We performed a comparison of the distribution of aggregated median prices as above between the group of stocks on the First Section of the market and the group of stocks on the other sections (Second Section, Mothers, JASDAQ Standard, and JASDAQ Growth).

- Stocks that were listed (or delisted) or changed markets during the period were excluded.

Sell/purchase ratio median distribution (Jan-Dec 2020)
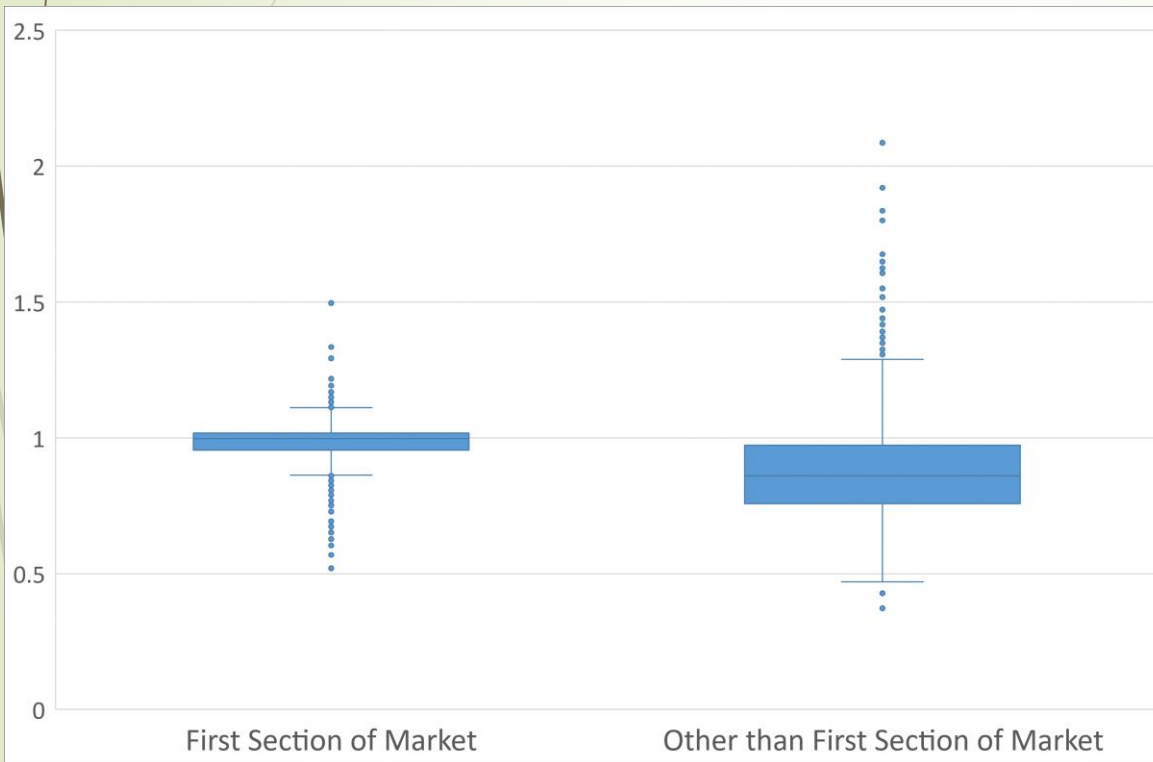
sell/purchase ratio
Median statistical
volume

	First Section	Other than First Section
Maximum value	1.496	2.086
75% points	1.018	0.973
Median value	0.998	0.861
25% points	0.955	0.758
Mininum value	0.520	0.372
Sample size	2133	1404
>1	1011	281
<1	1119	1116
=1	3	7

 There may have been greater restrictions on short selling outside the First Section*, where institutional investors hold a lower percentage of shares.
*TSE "Stock Distribution Survey" (https://www.jpx.co.jp/markets/statistics-equities/examination/01.html)

Summary

- Although market inefficiency on the short side only becomes apparent when the volume of sell orders is reduced, we could not confirm such a result was when increasing only the rate of increase in the fundamental price

  - Rather, the long-side breakdown tended to be slightly larger with a higher rate of fundamental price appreciation.

  The mechanism by which the short side breakdown increases is thought to be as follows:

  ① The number of sell orders presented within a certain range from the fundamental price becomes fewer

  ② There is a larger price increase in the best offer price when executed by a purchase order

  ③ The best quoted price deviates higher compared to the fundamental price.

- Whereas the sell/purchase ratio (median in 2020) for domestic stocks listed on the Tokyo Stock Exchange was around 1 in the First Section, there was a tendency for many stocks in the other market segments to be below 1.

Future Issues

- Simulation of patterns where there is a drop (including crash) in fundamental price

  - If the fundamental price (greatly) drops from the executed price, it is considered that short side market inefficiencies are significant

  - We should also analyze, from a threshold perspective, what extent of a decline in fundamental prices would result in a larger breakdown on the short side

- Analysis of how market inefficiencies on short side can be eliminated

  - In this study, we alluded to the process by which short side market inefficiencies become prominent, but we have not looked at a process to resolve this.

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Appendix A stylized fact reproduction results

➢ When changing parameter *q* and *d,* the reproduction results for fat tail and volatility clustering were not impaired.

Average statistical value from 50 times per parameter					
q	100	99	98	95	90
Kurtosis	15.13	14.11	14.09	14.08	14.09
Auto Correlation	Lag1	0.052	0.047	0.046	0.047	0.047
	Lag2	0.052	0.046	0.046	0.046	0.046
	Lag3	0.050	0.044	0.043	0.044	0.044
	Lag4	0.047	0.042	0.042	0.042	0.042
	Lag5	0.044	0.040	0.040	0.040	0.040

d		＋1	＋2	＋5	＋10
Kurtosis		15.12	15.13	15.12	15.13
Auto Correlation	Lag1	0.052	0.052	0.052	0.052
	Lag2	0.052	0.052	0.052	0.052
	Lag3	0.050	0.050	0.050	0.050
	Lag4	0.048	0.048	0.048	0.047
	Lag5	0.045	0.044	0.045	0.045

Note：Nominal distribution Kurtosis=0

This is also based on the return (excluding board construction period) whenever time *t* is passed.

Appendix B Transition in asymmetry of market inefficiencies over time

➢ With regard to parameter q, there was no indication of the asymmetry of market inefficiency being resolved over time.

Result of measuring the asymmetry of market inefficiencies whenever time 100,000 passed
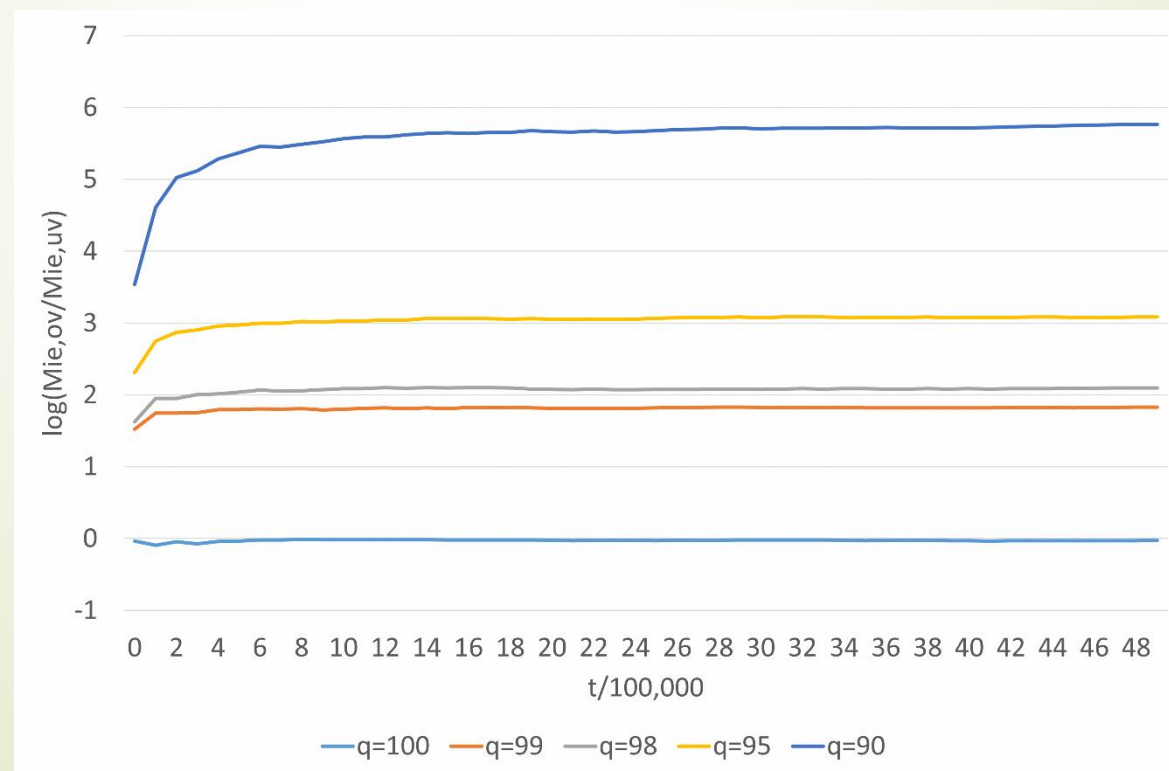
Appendix C-1 Sell/purchase ratio monthly transition (First section)

➢ As additional empirical analysis, we calculated the median value of sell/purchase ratio for each month in 2020, and investigated the transition.

**sell/purchase ratio median value statistical value
(First Section: every month)**

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Maximum value	1.753	1.890	1.573	1.776	1.895	2.013	1.760	1.866	2.147	3.241	2.053	1.681
75% points	1.005	1.011	1.048	1.029	1.032	1.050	1.063	1.036	1.021	1.066	1.038	1.035
Median value	0.954	0.965	1.009	0.993	0.992	1.010	1.019	0.991	0.978	1.019	0.998	0.997
25% points	0.878	0.887	0.953	0.949	0.937	0.961	0.968	0.929	0.915	0.969	0.941	0.945
Mininum value	0.324	0.233	0.413	0.272	0.111	0.266	0.382	0.278	0.237	0.189	0.165	0.147
Sample size	2158	2158	2159	2166	2169	2166	2167	2171	2170	2176	2174	2174
>1	584	672	1207	969	979	1224	1318	975	817	1355	1066	1037
<1	1573	1486	952	1194	1190	942	846	1196	1353	821	1107	1137
=1	1	0	0	3	0	0	3	0	0	0	1	0

Appendix C-2 Sell/purchase ratio monthly transition
(Other than First Section)

➢ In the markets other than the First Section, the median value of the sell/purchase ratio (within the analysis period) was less than 1 for a large number of stocks in the sample size.

sell/purchase ratio median value statistical volume (other than First Section monthly)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Maximum value	3.286	3.078	5.172	4.961	4.199	4.584	3.705	3.745	3.444	3.865	3.600	4.198
75% points	0.941	0.919	1.027	0.993	1.022	1.083	1.116	1.042	1.055	1.081	1.062	1.035
Median value	0.771	0.730	0.853	0.846	0.874	0.919	0.929	0.855	0.859	0.877	0.862	0.854
25% points	0.638	0.572	0.716	0.710	0.734	0.768	0.765	0.710	0.693	0.695	0.681	0.675
Mininum value	0.109	0.075	0.054	0.055	0.089	0.070	0.244	0.142	0.080	0.164	0.125	0.135
Sample size	1506	1499	1471	1493	1500	1493	1493	1499	1495	1500	1506	1499
>1	282	281	403	350	414	538	569	444	444	509	484	431
<1	1222	1218	1060	1135	1085	954	915	1053	1047	988	1017	1068
=1	2	0	8	8	1	1	9	2	4	3	5	0