Introduction to Japanese Government Bond (JGB) Futures[1]

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[1] We are grateful for the English editing to Miyu YAMANAKA.

1. Introduction

This paper aims to explain the basics of Japanese Government Bond (JGB) Futures. JGB Futures are widely used in Japan's fixed-income market but seem largely unfamiliar to those not involved in the market. Their daily trading volume is several trillion JPY, and the market is the most liquid market in JPY rate market. Through arbitrage transactions between JGB Futures and 7-year JGBs, the price information of JGB Futures spills over to the cash-JGB market, which also affects the entire yield curve structure of the JGB market. To realize the fair value parity of the JGB market, market participants always look for the arbitrage opportunities between the JGB Futures and cash JGB prices.

This paper shows practical examples that will help readers understand the usage of JGB Futures in a general context, including the fact that market participants assume the price of JGB Futures is highly correlated to that of cash-JGBs with a remaining term to maturity of 7 years ("7-year JGBs"). This means that investors holding JGB Futures position until their expiry will deliver or receive 7-year maturity JGBs at a delivery date.

This paper will also describe market practices widely known to JGB market players but rarely mentioned in existing textbooks and prepare the supplementary Boxes and Appendix for further understanding of JGB Futures in terms of mathematical points of view.

This paper is structured in the following way: Section 2 will outline the primary mechanism of futures contracts and Section 3 will expand on the characteristics and transactions of JGB Futures. Section 4 will explain their settlement methods, and Section 5 will go on to describe the cheapest-to-deliver (CTD) mechanism for JGBs. Section 6 will describe the relationship between JGB Futures and JGB repo markets. Finally, Section 7 will conclude the paper.[2]

2. What are JGB Futures?

2.1 A futures contract is one of the types of agreement to buy or sell a particular asset at a specified time in the future

A basic concept of futures contracts is to buy or sell a particular asset at a specified time in the future. Buying JGB Futures means making a contract to purchase cash-JGBs at a predetermined price at a specified time in the future. Imagine pre-ordering a book at a bookstore; you make a reservation now and when the pre-determined settlement time comes in the future, you pay the ordered amount and receive your book. Buying JGB Futures has a similar process; you make a reservation to buy JGBs in advance,

[2] The opinion expressed in this paper is the author's point of view, not the organization to which the author belongs.

and then make the payment and receive the cash-JGBs at the receiving cash-JGBs at the time of settlement.

Futures contracts are categorized as derivatives, as the instrument derives from the price movements of the product it is based upon (the underlying asset). In the case of JGB Futures, the underlying asset of the contract is cash-JGBs. JGB Futures are traded in Osaka Exchange, and the trading is highly concentrated on 10-year JGB Futures (hereafter, JGB Futures are assumed to be 10-year JGB Futures unless otherwise stated). Many futures contracts with assets such as equities, forex, and commodities are actively traded in financial markets worldwide. There are also many other derivative products in addition to futures contracts, such as forward contracts, swaps, and options.

As we described in the introduction, JGB Futures are linked to cash-JGBs with a remaining term to maturity of 7 years ("7-year JGBs"). This fact means that by buying (or selling) a JGB Futures, an investor will receive (or deliver) 7-year JGBs at a delivery date. As such, market participants assume that the price of JGB Futures is highly corelated to that of 7-year JGB. This mechanism will be described in Sections 4 and 5.

2.2 A major role of JGB Futures: providing a risk management tool

Before delving into the product specifications of JGB Futures, we will describe how market participants use JGB Futures to manage their market risk. The Ministry of Finance of Japan (the "MOF") issues cash-JGBs through an auction, and primary dealers, a group of institutions mainly comprised of investment banks, bid the JGBs for their customers. Since the Japanese government has enlarged the cumulative national debt, the size of JGBs issuance per auction is also immense, ranging from JPY one to two trillion (equivalent to around USD 10 to 20 billion). This means that each primary dealer has a possibility to purchase hundreds of billions worth of JPY in JGBs in one shot of the auction.[3] This suggests that these primary dealers could hold an enormous amount of JGBs as an inventory, so managing market risk of this inventory is essential for them in terms of securing business continuity.

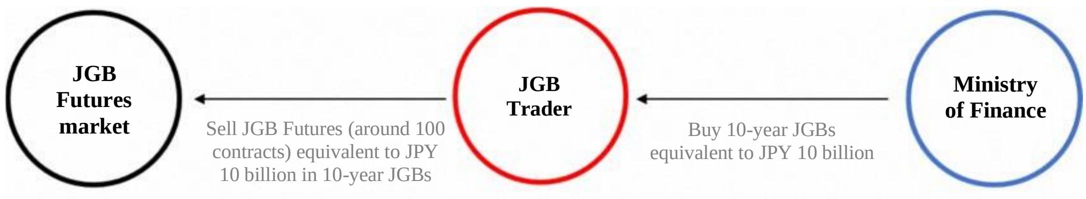
Holding inventory throughout business operations is a common practice in many industries, such as supermarkets. However, given their size and market volatility, holding JGBs as an inventory exposes primary dealers to significant market risk. For example, if prices of JGBs fall sharply immediately after an auction, primary dealers that won a bid at the auction may suffer huge losses. If the primary dealers are able to take a position that moves in the opposite direction of the price of the JGBs, they will be able to hedge the risks associated with the price fluctuations. JGB Futures enables market participants to make this position easily.

[3] In a practical world, the primary dealers, including brokerage firms, receive bid orders from end investors prior to the auction, so they do not always hold all the successful bid JGBs in inventory.

Some readers might think that short selling cash-JGBs is an easier way for hedging the market risk than selling JGB Futures. In the actual market, however, it is not easy to short cash-JGBs. A short cash-JGB position would require (1) borrowing JGBs from other market players and (2) selling them at the market. Investors can utilize a repurchase agreement ("repo") for lending and borrowing JGBs, but only a limited number of market players (comprised of brokerage firms that are market makers of the JGB market, top-ranked financial institutions, and major global investors) are able to access the repo market. The bottom line is that it is not easy to short sell cash-JGB as assumed. Taking a short position using JGBs futures enables investors to easily hedge against the price fluctuation risk which comes with changing interest rates, or the so-called interest rate risk, associated with JGBs.

Figure 1 shows a case in which a JGB trader uses JGB Futures to hedge their position. As described above, JGBs are issued by the MOF through auctions, and JGB traders act as primary dealers having an obligation to bid under the primary dealer rules. After the auction, JGB traders temporarily hold massive JGBs as an inventory, upon which they usually sell JGB Futures to hedge the interest rate risk associated with this position. Suppose a JGB trader has to temporarily store JPY 10 billion in 10-year JGBs as an inventory. In this case, the trader can hedge the interest rate risk by selling JGB Futures equivalent to the amount of risk exposure of the JGBs.

Figure 1: Example of market making for JGBs using JGB Futures - the case of bidding JGBs in auction

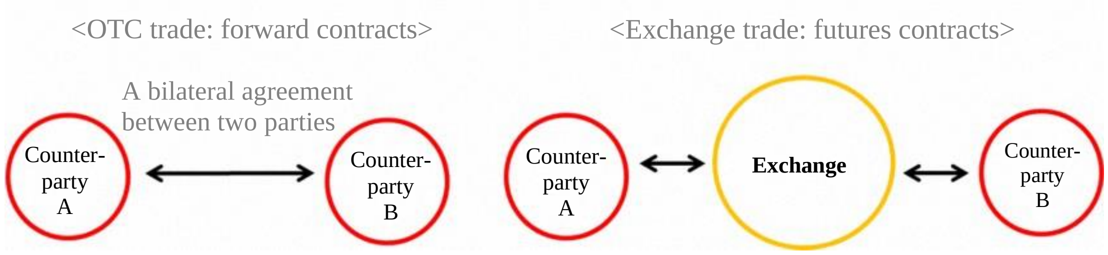


Note: For simplicity, this example assumes that one futures contract has the same amount of risk exposure as JPY 100 million of 10-year JGBs.

2.3 Difference from Forward contracts: JGB Futures are traded at the exchange

Many become confused regarding the difference between futures and forward contracts when beginning to learn about futures. Although a forward contract is a different type of derivative product, futures and forward contracts are similar in that they both agree to buy or sell a particular asset at a specified time in the future. The essential difference between them lies in their trading methods. A futures contract is traded on an exchange, while a forward contract is a bilateral agreement between counterparties. As Figure 2 shows, similar to listed equities, a futures contract is listed on and traded on an exchange such as Japan Exchange Group (JPX). On the other hand, a forward contract is agreed upon between two institutional investor counterparties, usually financial institutions (see the price mechanism of a forward contract in Box 2). Since a forward contract is made between two counterparties outside of an exchange, this type of market is called the Over-The-Counter (OTC) market.

Figure 2: Comparison between bilateral trading and exchange trading



People unfamiliar with JGB Futures sometimes get confused about their terminology or concept, which mainly stems from their exchange-trading mechanism. To list a futures contract on the exchange market, a futures contract needs to be standardized to meet as many investors' demands as possible. For JGB Futures, hypothetical standardized JGBs are set as an underlying asset rather than issued cash-JGBs, and JGB Futures investors deliver/receive the specific cash-JGBs chosen based on conversion rules at the delivery date. The tradable terms of futures contracts are also standardized. In the case of JGB Futures, the last trading and delivery dates are set for each quarter, and so the contracts are called "March Contract," "June Contract," "September Contract," and "December Contract," respectively.

Since JGB Futures are a form of the reservation to buy or sell JGBs in advance, the buyers or sellers are only to pay the amount on the delivery date, except for the collateral margin. Unlike forward contracts traded in the OTC market, investors can anonymously trade futures contracts with a counterparty. As such, a futures contract must ensure the security of transactions by evaluating all positions with mark-to-market and settling gains and losses of all counterparties daily. Through this mechanism, if the margin of a market participant becomes insufficient for the necessary amount of the day, the participant must deposit the additional margin by a specified time.

These pre-determined rules contributed to increasing the market liquidity of JGB Futures. As this margin mechanism allows investors to leverage their positions, the trade activities of JGB Futures tend to become more significant. As such, the JGB Futures market is now the most liquid in the JPY rate market, as described in the Introduction section. If a financial instrument is traded only once as day and closed at

JPY 100, the price only reflects just one investor’s view. However, if the price is determined through JPY trillions’ trade within that day, this price should contain the aggregated views of investors. As the JGB Futures are one of the most actively traded products in Japan’s fixed-income market, the information implicitly contained within the prices has important implications for cash-JGB prices. JGB Futures and cash-JGB prices interact through investors’ arbitrage transactions so that JGB Futures prices are considered the most critical benchmark for understanding the movements of the whole JGB market.

2.4 Characteristics of trading through exchanges and OTC markets

It was mentioned above that a notable characteristic of futures contracts is in their exchange-based nature, but it's worth noting that exchange trading and OTC trading both have their own advantages. Exchange trading has the benefit of higher market liquidity. In addition, exchange trading is seen as safer, as strict management rules ensure the robustness of transactions, including margin requirements, which come into play during situations of market turbulence.

On the other hand, OTC trading allows for flexibility in trading various types of securities or derivatives. It is unrealistic to list all of the fixed-income products on exchanges as there is a diverse range of issuers and a large variety of product specifications such as maturity structure. As for trading fixed-income products, investment banks usually hold them as inventory and quote prices to their customers. This method of transaction is not unique to fixed-income products: for example, the supermarkets where we get most of our daily necessities keeps a large inventory, where they store goods and sell one-on-one. In this context, it might even be that it is exchange trading which is the outlier.

One of the issues that caused the global financial crisis (GFC) in 2008 stems from the vulnerability of the OTC market, and monetary authorities tightened regulations for OTC trading, accordingly, including shortening the settlement period and encouraging derivatives transactions through a Central Clearing Party (CCP) that requires an appropriate amount of collateral. In this respect, OTC trading today has improved its credibility compared to before the financial crisis.

3. The product specifications of JGB Futures

The previous section provided a general overview of futures contracts. In the following section, we will describe the contract specifications of JGB Futures. As previously emphasized, it is important to note that many of the features described below can be interpreted as standardization to enable futures contracts to trade at exchanges.

10-year JGB Futures

One of the characteristics of the JGB Futures markets is that, for the most part, only 10-year JGB futures are actively traded. Table 1 shows the product specifications of 10-year JGB Futures. While 5-year and 20-year JGB Futures are listed on OSE, they have almost no transactions, which lies in stark contrast to the US Treasury futures market, where futures contracts with a wide range of maturity periods are actively traded. One possible reason to explain the difference is that the investor base for JGB Futures except for 10-year maturity is relatively smaller due to a lack of variety in investment strategies utilizing the JGB futures market, unlike in the U.S. treasury futures markets.

Table 1: Product specifications of 10-year JGB Futures (as of December 2022)

Trading started at	October 19, 1985
Contract	Standardized 6%, 10-year JGBs
Deliverable grades	Interest-bearing 10-year JGBs with a remaining term to maturity of at least 7 years, but not more than 11 years
Contract months	Quarterly contracts (March, June, September, December) listed for 3 quarters
Last trading date	5th business day prior to each delivery date (20th day of each contract month, move-down the date when it is not the business day).
Contract unit	Face value at maturity of JPY100 million
Minimum tick	JPY 0.01 per JPY 100 face value
Margin	Calculated by using SPAN®
Settlement methods	1. Close out the position 2. Physical delivery at the final settlement

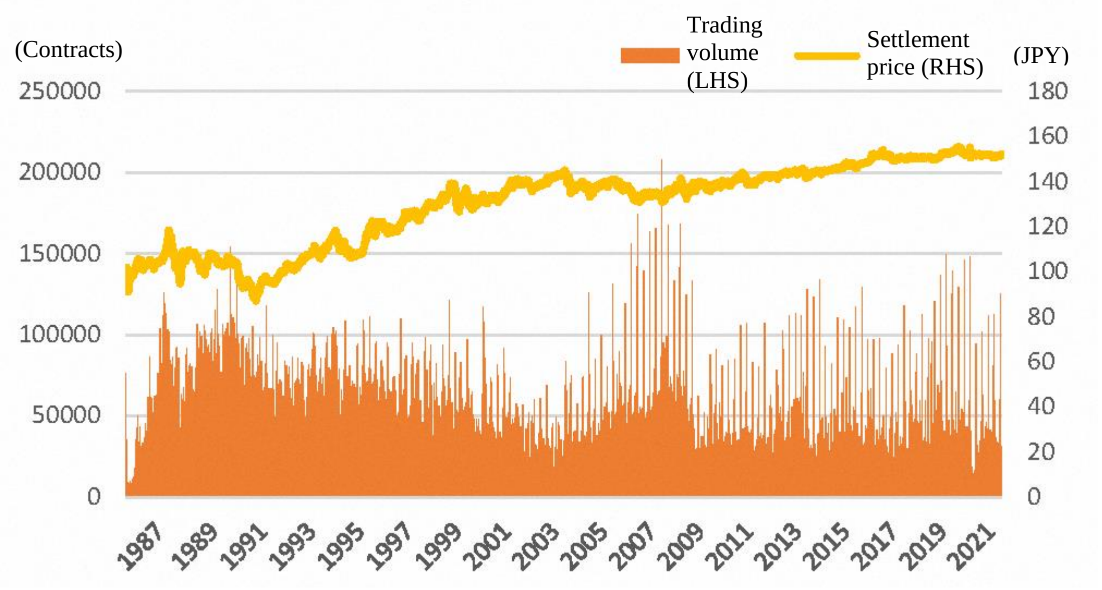
Source: Japan Exchange Group

Standardization

As described above, JGB Futures are standardized contracts for buying and selling underlying JGBs for future delivery. For 10-year JGB Futures, the standardized underlying cash-JGBs have a 6% coupon rate and a remaining term to maturity of 10 years. At the delivery of the 10-year JGB Futures, the 10-year coupon-bearing cash-JGBs with a remaining term to maturity of between 7 and 11 years are delivered, of which delivery is based on the “Conversion Factor (“CF”).” The concept of the CF will be explained in Box 1, while its mathematical derivation is shown in Appendix.

Figure 3 shows the changes in settlement prices and the trading volume of 10-year JGB Futures. The face value of JGBs at maturity is JPY 100 in commercial practice, and recently 10-year JGB Futures have been priced much higher, even hitting the JPY 150 level. These higher futures prices compared with the face value reflects the difference between around 0% of the lower coupon rate of cash-JGBs and 6% of the standardized coupon rate of 10-year JGB Futures.

Figure 3: Settlement prices and trading volume of 10-year JGB Futures



Source: Japan Exchange Group

Trading hours

The daytime trading hours at Osaka Exchange are divided into two sessions: a Morning Day Session, from 8:45 a.m. to 11:02 a.m. JST, and an Afternoon Day Session, from 12:30 p.m. to 3:02 p.m. JST. After these daytime sessions, a Night Session starts from 3:30 p.m. to 6:00 a.m. JST the following day. Transactions are concentrated in the Morning and Afternoon Day Sessions. A closing auction is conducted after the end of continuous trading hours of each session, and the closing auction period is the last two minutes of the Morning and Afternoon sessions and the last five minutes of the Night session. The closing price at the closing auction is determined by the so-called *itayose* method[4].

Contract unit

The contract size of a single JGB Futures contract is JPY 100 million, suggesting that buying or selling one unit of JGB Futures contacts is equivalent to holding long or short position amounts to JPY 100 million worth of JGB Futures. This amount of JPY 100 million is referred to as "notional contract value (or notional amount)." As shown in Figure 3, the current average daily trading volume of 10-year JGB Futures is about 40,000 contracts or JPY 4 trillion on a contract notional value basis.

[4] For more details about the *itayose* method, refer to JPX's website from the link below.
https://www.jpx.co.jp/english/derivatives/rules/trading-methods/index.html

Futures margin

To buy one unit of JGB Futures, equivalent to the notional amount of JPY 100 million, investors do not need to prepare JPY 100 million. Instead, investors only need to pay the futures margin required, calculated according to their positions. Futures margins are measured by a calculation methodology called SPAN®, or the Standard Portfolio Analysis of Risk, and its fundamental objective is to cover future losses arising from current positions by using the expected price volatility (implied volatility) of the futures contracts.[5] Suppose a futures margin of JPY 1 million is required to buy one JGB Futures contract. In this case, an investor can take the position of JPY 100 million in terms of the notional value of JGB Futures with collateral of JPY 1 million. This suggests that the investor can trade with 100 times leverage (100 million/1 million) using JGB Futures. Since the futures margin is calculated based on parameters such as expected price volatility, the level of leverage depends on the market conditions at that time. Note that the futures margin measured by SPAN® is the minimum required amount between a CCP and a trading participant, and investors can reduce the level of leverage by increasing the deposited futures margin amount.

Contract months and a final settlement date for delivery

The month in which futures contracts expire is called the contract month. The delivery date for JGB Futures is the 20th day of each contract month (or if it falls on a business holiday, the following day), and the last trading date of the contract month is the 5th business day prior to the delivery date. As mentioned above, JGB Futures have quarterly contract months in March, June, September, and December, and the contracts of the upcoming three can be traded. For example, as of January 2021, the listed contract months of JGB Futures are March-2021, June-2021, and September-2021s. When the March-2021 contract month is expired, a new contract month, the December-2021 contract month, will be launched.

In the JGB Futures market, a contract month whose last trading date is closest to the current date, the March-2021 contract in the above case, tends to be the most actively traded, and such a contract month is called an active month contract. Investors typically trade the nearest contract month: the March-2021 contract month in the above case. As the last trading date for this contract month approaches, investors roll their positions to the following nearby contract month (the June-2021 contract month in the above case). As such, one contract month is only actively traded for three months in practice, although theoretically it can be traded for nine months. As shown in Figure 3, when using time series price data of JGB Futures contracts, prices of the nearest active contract months are usually combined.

[5] Expected price volatility of 10-year JGB Futures is calculated based on the implied volatility of Options on JGB Futures.

Circuit breaker

OSE limits the maximum price fluctuation range of JGB Futures by using a circuit breaker mechanism. Circuit breakers are a mechanism which curb excessive price movements by forcibly halting trading on an exchange for a certain period when market prices fluctuate beyond a ceiling level. The circuit breakers for JGB Futures suspend trading for at least 10 minutes when the market price hits the limit range prices. The circuit breaker mechanism was originally introduced in response to the U.S. market crash of 1987, or the "Black Monday."

4. Physical delivery and a Conversion factor

4.1 Physical delivery or cash settlement: physical delivery for JGB futures

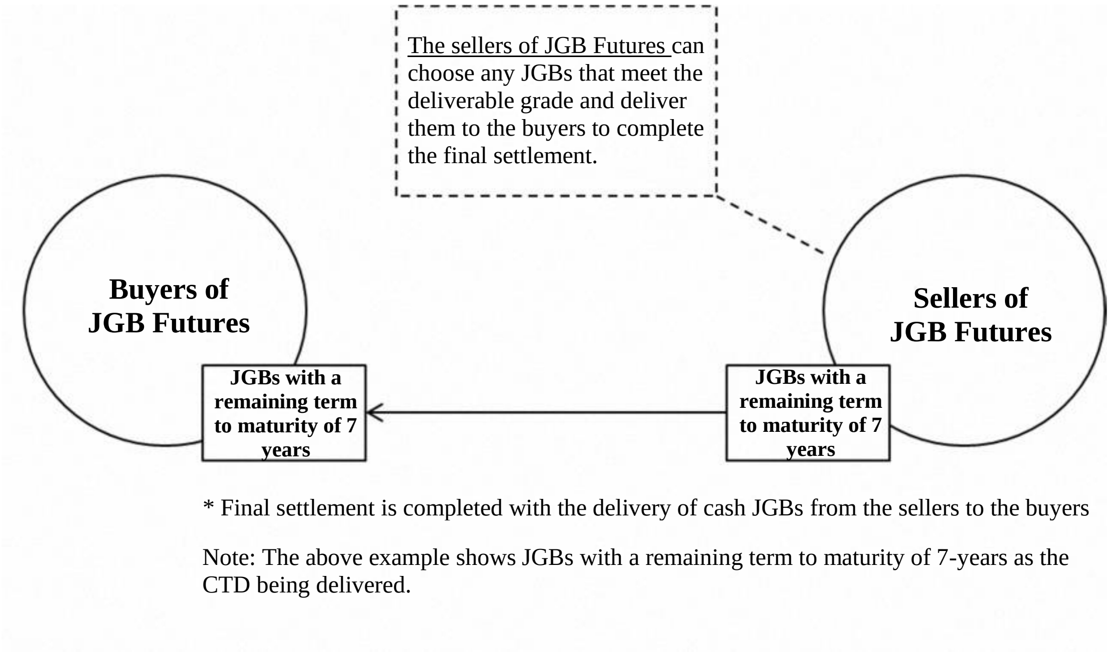
The settlement method of JGB Futures consists of "physical delivery" and "closing out a position." Any remaining positions of JGB Futures by the last trading date will be settled by the delivery of cash-JGBs, in a transaction called the "physical delivery" of JGBs. Also, a buyer or seller can avoid physical delivery for the final settlement by off-setting trade before the last trading day. This cash-settlement method is called as "close out."

As for physical settlement, there is a specific range of requirements on the JGBs to be delivered, set as "interest-bearing 10-year JGBs with a remaining term to maturity of over 7 years but under 11 years." This range is usually called a delivery basket. As shown in Figure 4, the sellers of the JGB Futures can select any JGBs with a remaining term to maturity of 7 to 11 years for delivery. The eligible JGBs for physical delivery are called "deliverable grade" bonds. In the case of JGB Futures, sellers have the option to select the JGBs among the baskets, which is usually called the "deliverable option." One of the main background reasons for introducing a delivery basket mechanism is to avoid a short squeeze on the market.[6]

Fixed-income futures tend to apply physical delivery as a final settlement method, while other types of futures, particularly equity index futures, usually adopt cash settlement, which transfers a final net-cash position, as a final settlement method.

[6] See Tuckman and Serrat (2011).

Figure 4: An example of physical delivery settlement in JGB Futures contracts



4.2 Conversion Factor (CF)

In the JGB Futures market, investors trade hypothetical standardized JGBs; in the case of 10-year JGB Futures, underlying JGBs are defined as 6% interest-bearing 10-year JGBs with a remaining term to maturity of 10 years. On the delivery date, however, the sellers and buyers of JGB Futures have to exchange the actual cash JGBs among a delivery basket ranging from 7 to 11-year JGBs. To compute a fair price of the actual delivered JGBs, the sellers and buyers rely on the Conversion Factor ("CF"). The CF is a complex mathematical formula, but its intuition is the factor which converts the price of standardized JGBs set for JGB Futures into the price of issued JGBs that are eligible for delivery. The exact definition and derivation of the CF are described in Box 1.

The price for the physical delivery of JGB Futures is determined by multiplying the JGB Futures price by the CF as below:

$$\text{Delivery price} = \text{JGB Futures price} \times \text{CF}$$

For example, if the CF of a JGB with a remaining term to maturity of 7 years is 0.7 and the JGB Futures price is JPY 150, then JPY 105 (or CF 0.7 × JPY 150) is the delivery price of the 7-year JGB. As described in BOX 1, the level of the CF depends on the coupon rate and the remaining term to maturity of the interest-bearing JGBs to be delivered. Hence, the CF values differ for each eligible JGB.

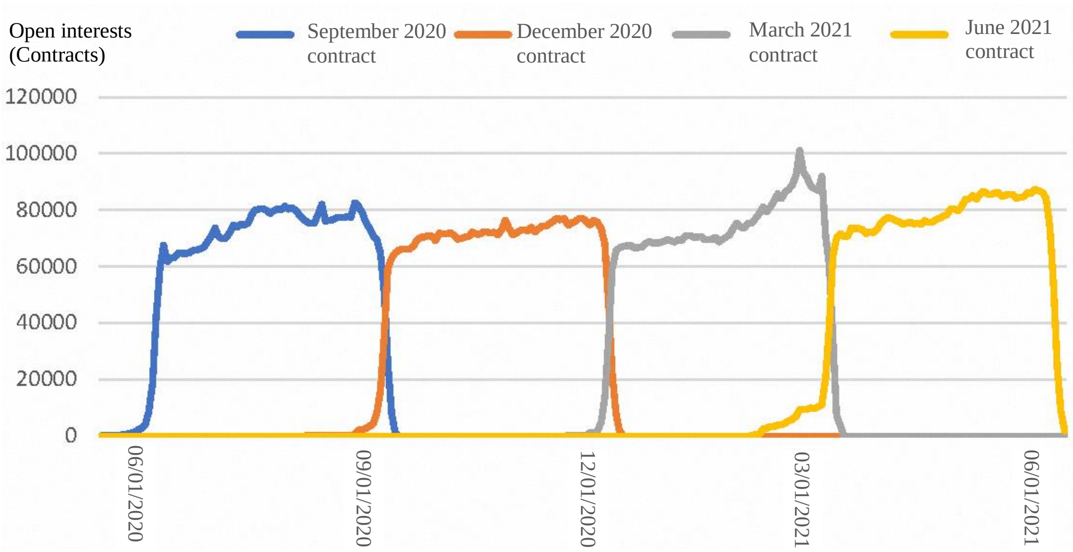
As described in Section 2, JGB Futures are linked to cash-JGBs with a remaining term to maturity of 7 years, and by buying or selling JGB Futures, the investors receive or deliver 7-year JGBs at a delivery date. This is because under current market conditions, the delivery cost for 7-year JGBs becomes the cheapest among eligible JGBs when calculated by the formula of 10-year JGB Futures price × CF. The JGB with the lowest delivery value is referred to as the "Cheapest to Deliver (CTD)," and this detailed mechanism is explained in the following section. Box 1 explains why the price of the eligible JGBs for delivery is determined by multiplying the JGB Futures price by the CF and why the CF is likely to be around 0.7 within the current low-interest-rate environment. For the actual value of the CFs, please refer to the JPX website.[7]

4.3 Closing out positions and calendar spread trading between contract months

As the last trading day of the JGB Futures contracts approaches, investors should choose whether to close out positions, receive/deliver JGBs through the physical delivery process, or roll over their positions to the next contract month. Figure 5 shows the changes in open interest of JGB Futures with different contract months since June 2020. Open interest means the total number of all unsettled futures contracts and intuitively represents the total number of the whole size of the market. As figure 5 shows, the amount of open interest gradually declines by the last trading day of its respective contract month. This is because investors close out their positions by taking opposite positions before the last trading day to avoid delivering the underlying assets. This means the buyers and sellers can choose "close-out trades" instead of "physical delivery" for the final settlement. When holding a selling position on JGB Futures, an investor can close out the position by buying back the same number of futures contracts before the last trading date, which allows the investor to avoid physical settlement.

[7] https://www.jpx.co.jp/english/derivatives/products/jgb/jgb-futures/03.html

Figure 5: Changes in open interest of each JGB Futures contract month



Source: Japan Exchange Group

When the last trading date is approaching and an investor wishes to keep their short positions for hedging purposes, the investor can roll over the short positions by simultaneously buying back futures of the holding contract month to close out the position while selling futures of the following contract month. This type of trading is called calendar spread trading between contract months, or simply calendar trading. Hull (2017) classify this strategy as Stack and Roll. When investors try to roll over short position of futures, the investors will trade as below:[8]

$$Timer\ t_1: Short\ futures\ contract\ 1$$
$$Timer\ t_2: Close\ out\ futures\ contract\ 1, short\ futures\ contract\ 2$$
$$Timer\ t_3: Close\ out\ futures\ contract\ 2, short\ futures\ contract\ 3$$

JGB Futures trading began in 1985, and calendar spread trading was introduced in 2000. A futures contract with the nearest expiry date is normally called "front month," and futures with the second-nearest contract months are called "back month." For an investor to avoid the physical delivery of cash-JGBs while maintaining a short position, they need to simultaneously buy back the front month and sell the closest back month. For accurate rollover, the front month and the nearest back month should be traded at the exact same timing, and calendar spread trading is the mechanism which enables these transactions.

[8] This description refers to Hull (2017) and see the reference for details.

The trading volume of calendar spread increases toward the last trading day of a front month contract, which indicates the increasing demand for rolling over positions to the second nearest month.

BOX 1: Derivation of the Conversion Factor[9]

1. Definition and interpretation of the Conversion Factor

As mentioned in the previous sections, trading JGB Futures entails trading a standardized futures contract with underlying hypothetical cash-JGBs. As such, there needs to be an adjustment to convert the standardized JGB price to the issued JGB price to be delivered in the final settlement. This adjustment is made through using the Conversion Factor ("CF"). The fundamental logic behind the CF is first to calculate the present values ("PVs") of the standardized JGBs and deliverable JGBs, assuming a 6% flat yield curve and then comparing them. Specifically, we equalize the PVs by multiplying the CF, as below:

$$\text{The present value of the deliverable JGB} = \text{The present value of the standardized JGB} \times CF$$

In other words, CF is defined as follows accordingly:

$$\text{CF} = \frac{\text{The present value of the deliverable JGB}}{\text{The present value of the standardized JGB}}$$

The present value of the standardized JGB should be JPY 100 as a 6% coupon-bearing JGB is discounted with a 6% flat yield curve. Therefore, the above equation can be expressed as follows:

$$\text{CF} = \frac{\text{The present value of the deliverable JGB}}{100}$$

This equation means that the CF is to divide the deliverable JGBs, discounted with a flat curve of 6% by 100. If you discount the deliverable JGB by the 6% flat curve by using financial tools such as Bloomberg, and divide the price by 100, you will obtain the rough value of the CF[10]. Under the current environment, where the coupon rates of deliverable JGBs are extremely low, the CF value becomes around 0.7. This means that investing in JGBs with low coupon rates and realizing a compound yield of 6% needs a low JGB price of around JPY 70, and the CF value of 0.7 is derived since the PV of the deliverable JGB

[9] This Box refers to Hull (2017) and other literature. Ryo Ishida and Hajime Fujiwara supported the description of this Box.
[10] JPX publishes the exact value of the CF on its website (https://www.jpx.co.jp/english/derivatives/products/jgb/jgb-futures/03.html).

(roughly JPY 70) is divided by JPY 100. The CF value becomes lower according to the lower coupon rate of deliverable JGBs as the lower PV of deliverable JGB price is required to achieve the compound yield of 6%.

The same reasoning can explain why the CF of shorter-maturity bonds (e.g., 7-year JGBs) tends to be larger than longer-maturity bonds (e.g., 10-year JGBs). As mentioned before, the prices of JGBs realizing a compound yield of 6% should be lower than JPY 100 within the current low-interest rate environment. Here we should emphasize that assuming the same coupon rates, the PV of shorter-maturity bond is smaller than the PV of longer-maturity bonds as a longer duration makes the bonds more sensitive to interest rates changes. In other words, the degree of the discount for longer-maturity bonds such as 10-year JGBs should be larger that of shorter-maturity bonds such as 7-year JGBs, to realize a compound yield of 6%. Therefore, the CF of shorter (longer) maturity bond is larger (smaller) than the one of longer (shorter) maturity bond if the yield is lower than 6%. This explains why 7-year JGB becomes the CTD of JGB Futures.

This box defines the CF as "the present value of the deliverable JGB = the present value of the standardized JGB×CF," assuming a 6% flat yield curve environment. Suppose this CF is fixed as given, and this equation is still valid under the ACTUAL (NOT 6%) yield curve. In this case, the PV of the standardized JGB under the actual yield curve becomes equal to the price of JGB Futures traded at the market (the JGB Futures price). Therefore, the PV of the deliverable JGB under the actual yield curve can be represented as "JGB Futures price × CF."

2. Derivation of the Conversion Factor

The CF of JGB Futures is defined on JPX's website as follows:[11]

$$\mathrm{CF} = \frac{\frac{a}{0.06} \times \left((1 + \frac{0.06}{2})^{b} - 1 \right) + 100}{(1 + \frac{0.06}{2})^{\frac{c}{6}} \times 100} - \frac{a(6-d)}{1200} \qquad \cdots (1)$$

In this equation for computing deliverable JGBs, *a* is the amount of interest payable per year, *b* is the number of payments from the delivery day to the redemption date, *c* is the number of months from delivery day to the redemption date, and *d* is the number of months from delivery day to the next coupon payment date.

[11] https://www.jpx.co.jp/english/derivatives/products/jgb/jgb-futures/tvdivq0000003ncd-att/formula.pdf

To grasp the meaning of this equation, let us simplify Equation (1) as below:

$$\mathrm{CF} = \frac{\frac{a}{0.06} \times \left((1 + \frac{0.06}{2})^{\frac{c}{6}} - 1 \right) + 100}{(1 + \frac{0.06}{2})^{\frac{c}{6}} \times 100} \quad \cdots (2)$$

The 100 in the denominator of Equation (2) corresponds to the PV of the standardized JGB. Since CF is equal to the present value of the deliverable JGB/100 by definition, the PV of the deliverable JGB is as follows

$$\frac{\frac{a}{0.06} \times \left((1 + \frac{0.06}{2})^{\frac{c}{6}} - 1 \right) + 100}{(1 + \frac{0.06}{2})^{\frac{c}{6}}} \quad \cdots (3)$$

Now, we discount the cash flows generated by the deliverable JGBs by the 6% flat yield curve under JGB Futures' assumption. The coupon rate of the deliverable JGBs is *a*, the coupon is paid at the rate of *a/2* every six months, and the term to maturity is *N* years. We can compute the PV of the deliverable JGBs as P below by discounting the future cash flows by a flat yield curve of 6%.

$$P = \frac{a/2}{1 + 0.06/2} + \frac{a/2}{(1 + 0.06/2)^2} + \cdots + \frac{a/2}{(1 + 0.06/2)^{2N}} + \frac{100}{(1 + 0.06/2)^{2N}}$$

Please note that the future cash flows consist of the principal amount of 100 and a 6-month coupon rate of *a/2*.

By multiplying the above by $(1 + \frac{0.06}{2})^{2N}$, we can obtain the equation as follows:

$$P(1 + \frac{0.06}{2})^{2N} = \frac{a}{2}(1 + \frac{0.06}{2})^{2N-1} + \frac{a}{2}(1 + \frac{0.06}{2})^{2N-2} + \cdots + \frac{a}{2} + 100 \quad \cdots (4)$$

By using geometric progression, equation (4) can be also transformed as below:

$$P(1 + \frac{0.06}{2})^{\frac{c}{6}} = \frac{a}{0.06}\left((1 + \frac{0.06}{2})^{\frac{c}{6}} - 1 \right) + 100$$

Note that *c* is calculated monthly based on JPX’s definition (12*N* = c).

If we solve the above equation for *P*, we get equation (3), which successfully shows the formula published on JPX's website. We simplify the formula of the CF here, so please see the exact derivation in Appendix.[12]

We dropped the second equation (1) term to simplify the explanation. The intuition of a(6－d)/1200 is an adjustment component for accrued interest. The timing of redemption and interest payments for 10-year coupon-bearing JGBs are set in March, June, September, and December, similar to the futures contracts. Therefore, there are cases where there is accrued interest, and cases where there is not. To make it easier to understand $a(6-d)/1200$, let us assume that coupon payment takes place once a year, instead of once every half a year. In this case, the second term becomes $a\,(12-d)/1200$. Given d represents the period (months) up to the next interest payment date, when $d = 12$, no interest is accrued because the interest payment takes place a year later. Therefore, the second term of Equation (1) will be deleted. On the other hand, if $d = 0$, then $12a/1200 = a/100$. In this equation, the value of the one-year coupon of the deliverable issue is divided by the present value of the standardized JGB. Thus, the component $a(6-d)/1200$ can be interpreted as an effective adjustment for accrued interest.

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5. Arbitrage between the cash JGB and JGB Futures prices

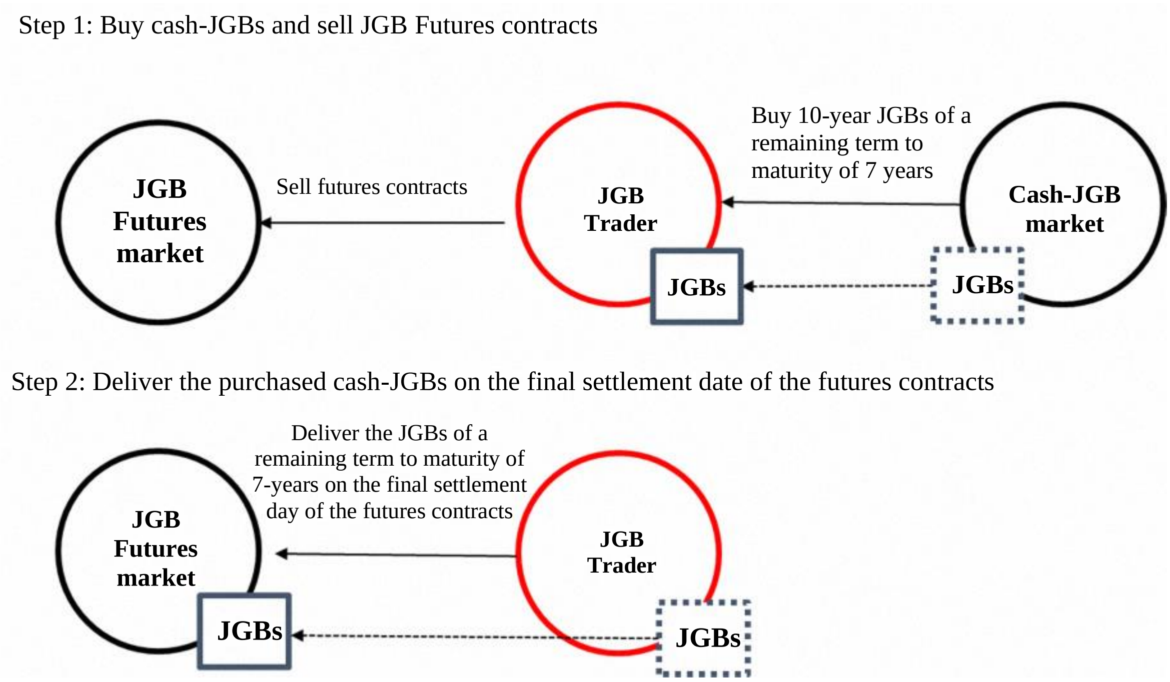
5.1 Arbitrage between cash-JGBs and JGB Futures prices: Basis trading

In the previous sections, we addressed a theoretical background on why 7-year JGBs are delivered at a delivery date among investors holding the JGB Futures positions. Based on that, this section will focus on the arbitrage mechanism between cash-JGBs and JGB Futures, a fundamental concept among institutional investors in the JGB market.

Given the linkage between 7-year JGBs and JGB Futures, an investor must deliver 7-year JGBs on the delivery date if the investor keeps short positions until the end of the last trading day. Suppose an investor purchases 7-year JGBs at the OTC market to prepare the delivery simultaneously as selling JGB Futures (see Step 1 in Figure 6). Then, on the delivery date of the JGB Futures contract, the investor will deliver the 7-year JGBs that you have already purchased to settle the futures contract (Step 2 in Figure 6). This is the arbitrage transaction between cash-JGBs and JGB Futures, so-called basis trading.

[12] The case where c = 6b is being considered.

Figure 6: Arbitrage between the cash-JGBs and JGB Futures prices



Note that at the physical delivery throughout this arbitrage transaction, 7-year JGBs are not delivered at the "JGB Futures price" but at the "JGB Futures price × CF." Again, the standardized JGBs are assumed as underlying assets of JGB Futures, so the standardized JGB prices need to be converted to the delivery price of actually issued JGBs through the CF. The delivery price is calculated as "Delivery price = JGB Futures price × CF." In this context, arbitrage opportunities are measured by comparing (1) JGB Futures price × CF and (2) the price of the cash-JGBs (7-year JGBs).

To consider the actual arbitrage transaction, assume selling (1) one unit of JGB Futures contract (notional amount of JPY 100 million), which expires in one month, and buying JPY 100 million worth of 7-year JGBs. If the JGB Futures price × CF is JPY 100, whereas the 7-year JGB price is JPY 99, an investor will earn a profit of JPY 1 (or JPY 1 million in face value) by selling the JGB Futures and simultaneously buying the 7-year JGB at JPY 99. In this case, the investor delivers 7-year JGBs at JPY 100 at the delivery date one month later. An arbitrage opportunity arises when there is a significant difference between JGB Futures price × CF and the 7-year JGB price. On the other hand, if this price difference is slight or zero, we can say that the market is efficient enough not to realize such arbitrage opportunities.

5.2 Gross basis and Net basis

In the JGB Futures market, the difference between the cash-JGB price and the JGB Futures price × CF is typically called "gross basis" shown as below[13]:

$$\text{Gross basis} = \text{Cash JGB price} - \underbrace{\text{JGB Futures price} \times \text{CF}}_{\text{The delivery price of JGBs calculated through the JBG Futures price}}$$

However, the actual arbitrage transaction should consider (1) a cost from fundraising and (2) a return from coupon payment. An investor has to raise the funding for a month to purchase 7-year JGBs in practice, and the cost is called repo costs. In contrast, the investor can earn coupon income by holding 7-year JGBs. For the realistic arbitrage world, the repo cost and coupon payment are incorporated as below:

$$\text{Net basis} = (\text{Cash JGB price} - \underbrace{(\text{Coupon income} - \text{Repo cost})}_{\text{Carrying costs}}) - \text{JGB Futures price} \times \text{CF}$$

Market participants usually call this concept as net basis, and "Coupon income - Repo cost" are called "carrying costs" or "carry" (see Tuckman and Serrat 2011).

As described in Box 2, the cash-JGB price adjusted for carrying costs is equivalent to the JGB forward price, so a net basis can be interpreted as an arbitrage position between JGB Futures contracts and JGB forward contracts. As mentioned in Section 2.3, one of the main differences between futures contracts and forward contracts is that futures contracts are traded on exchanges, whereas forward contracts are made in the form of a bilateral agreement between contract parties in the OTC market. Essentially, however, both contracts are similar in that both are the agreements to buy or sell a particular asset at a specified time in the future.

$$\text{Net basis} = \underbrace{\text{JGBs forward price}}_{\text{Cash JGBs price} - \text{Carrying cost}} - \text{JGB Futures price} \times \text{CF}$$

---

[13] The gross basis is defined as "cash JGBs price - JGB Futures price × CF," but in terms of the discrepancy between the prices of cash and futures contracts, the basic concept remains unchanged even if it is defined as "JGB Futures price × CF - cash JGB price." However, according to business practice, the gross basis is defined as "cash JGBs price - JGB Futures price × CF."

In practice, taking a long cash position and a short futures position are respectively called "long the basis," while taking a short cash position and a long futures position are called "short the basis." If an investor takes a long (or short) basis position, the investor will make a profit if the gross basis as defined above rises (or falls). Suppose that the gross and net basis currently deviate significantly from zero in a negative direction, and the investor expects the gaps to converge to zero by the delivery date of the futures contracts. In this case, taking a long basis position will make an investor profit when this expected price convergence occurs. On the other hand, if the gross and net basis deviates from zero to positive, there is an arbitrage opportunity by taking a short basis position.

This type of arbitrage is called a "relative value" strategy and is widely used as a typical investment strategy in the fixed-income markets. A trader can simultaneously order the cash-JGBs and JGB Futures as a packaged transaction, so-called asset swap trading.

5.3 Cheapest-to-deliver (CTD)

In the previous sections, we regarded the situation that investors of JGB Futures receive or deliver 7-year JGBs at a delivery date as a given condition. This section will describe details about why 7-year JGBs become the Cheapest to Deliver ("CTD") of JGB Futures. As sellers of 10-year JGB Futures can deliver any cash-JGBs with a remaining term to maturity of 7 to 11 years, they rationally choose the cash-JGBs that give them the largest profit. Suppose an investor sold 10-year JGB Futures and had not closed out the contracts by the last trading date. In this case, the investor is obliged to deliver cash-JGBs to the buyers on the delivery date. If the investor does not have JGBs with a remaining term to maturity of 7-11 years in their portfolio, the investor must purchase the JGBs for delivery through investment banks, for example, before the delivery date. The purchase cost to purchase these cash-JGB deliverables correspond to the delivery costs for the investor.

Let us consider, on the other hand, the revenue the investor will gain. As the delivery price is calculated by JGB Futures price × CF, the revenue generated from the delivery of the JGBs is also JGB Futures price × CF. Based on the cost and revenue structure outlined above, the selling investor has an incentive to deliver the cash-JGBs that will generate the largest profit based on the following formula among the deliverable cash-JGBs with a remaining term to maturity of 7 to 11 years:

$$\underbrace{\text{JGB Futures price} \times \text{CF}}_{\text{Revenues from the delivery}} - \underbrace{\text{Cash-JGB price}}_{\text{Costs associated with the delivery}} \quad \cdots \quad (*)$$

If the cash-JGBs with a remaining term to maturity of 7 to 11 years are traded on the OTC market at almost the same prices, then selecting the cash-JGB with the bigger CF will be key to generating the largest profit. Therefore, the JGBs with the biggest CF will be the most profitable JGBs for delivery, but we should note here that if the coupon rate is 6% or below, the shorter-maturity bonds have larger CFs as long the coupon rates of cash-JGBs are at the same condition (See Box 1 for the details of this mechanism). Therefore, delivering shorter-maturity JGBs becomes advantageous to a seller of 10-year JGB Futures who need to deliver cash-JGBs. As coupon rates in Japan have been lower than 6% for a long time, 7-year JGBs (the shortest term to maturity among deliverable JGBs) have been continuously chosen as the deliverables for 10-year JGB Futures contracts. Since 7-year JGBs are the most profitable choice among the delivery baskets, 7-year JGBs are called the “cheapest-to-deliver (CTD),” representing the lowest delivery cost.

Because this discussion is, in the end, a comparison between the cash JGB prices and JGB Futures price × CF, market participants use the gross and net basis to compute which JGBs among the deliverable basket should become the CTD. As the gross basis of JGBs with a remaining term to maturity of 7 to 11 years is defined as “Cash-JGB price - JGB Futures price × CF”, the seller of JGB Futures will generate more profit in delivering cash-JGBs with a smaller gross basis. In actual derivative transactions, however, the coupon income and repo cost must be taken into account, so market participants normally use net basis (JGB forward price－JGB Futures price × CF) to compute the CTD.

Box 2: Pricing a Forward contract

The basic concept of forward contracts was mentioned in Section 2.3, and this Box will explain their pricing mechanism. For example, suppose a JGB trader receives a customer’s order to purchase JGBs for delivery a week later. In this case, the JGB trader has to price this forward contract for this customer, and the simplest way to handle this order is (1) to purchase the JGBs right now, (2) hold the JGBs as inventory, and (3) deliver the JGBs a week later to the customer. Therefore, the fair price of this forward contract should be determined as the current JGB price with the holding cost as an inventory.

The model for pricing the forward price in this way is called the “cost of carry.” Assume pricing a JGB forward contract due after N days. Like the earlier discussion, you will now purchase cash JGBs and hold them in inventory for N days bearing the financing cost (or the repo cost) for the purchase. During the term, you will enjoy the coupon income generated by holding the JGBs, which can be deducted from the repo cost. In this context, the JGB forward price is defined as follows:

$$\text{JGB forward price} = \text{Cash JGB price} + (\text{Repo cost} - \text{Coupon income})$$

As explained in Section 5.2, the price difference between the coupon income and the repo cost is called carrying cost or carry, so the JGB forward price is described as the cash JGB price minus the carry.

Box 3: Interest Rate Risk Exposures of JGB Futures

Interest rate risk refers to the risk of asset price fluctuations resulting from a change in interest rates. In practice, factors such as duration are widely used in measuring the interest-rate risk of fixed-income products. In addition, JGB Futures are also utilized for measuring the interest-rate risk: calculating the JGBs risk amount based on how many futures contracts are equivalent to the risk.

Based on basis point value (BPV) or DV01, we will calculate the interest-rate risk associated with JGB Futures under the assumption of the delivery of JGBs with a remaining term to maturity of 7 years. BPV or DV01 indicates the degree of change in price in response to a change of 1 basis point (0.01%) in interest rates.[14] Let's assume *P* is the JGB Futures price, *CF* is the conversion factor, and *CTD* is the 7-year JGB price; then, we can obtain $P \times CF = CTD$. We can also calculate DV01 of JGB Futures by differentiating the JGB Futures price ($P = 1/CF \times CTD$) with the interest rate (r):

$$\text{DV01 of JGB Futures} = \frac{\Delta P}{\Delta r} = \frac{1}{CF}\frac{\Delta CTD}{\Delta r} \times 0.01\%$$

Since $\Delta\ CTD\ /\Delta r$ is the DV01 of the 7-year JGBs, the risk amount associated with a 1-basis point change in interest rate is approximately JPY 0.07 (per JPY 100). The DV01 of JGB Futures ($\Delta P/\Delta r$) can be calculated as the $DV01\ of\ 7\text{-}year\ JGB/CF$. Under the current market conditions, the value of CF is approximately 0.7, therefore the DV01 of the JGB Futures becomes approximately JPY 0.1 to 0.11 (per JPY 100). This means that DV01 of JGB Futures is close to DV01 of 10-year JGBs. Note that while the price movement of JGB Futures is linked to 7-year JGBs, the changes in JGB Futures price are amplified by 1/CF. As a result, the DV01 of the JGB Futures is roughly equivalent to the DV01 of 10-year JGBs.

Lastly, we will see how many JGB Futures contracts correspond to 20-year JGBs in terms of the risk amount. When the DV01 (per JPY 100) of a JGB Futures contract is JPY 0.11, and that of 20-year JGBs is JPY 0.19,[15] the risk amount of 20-year JGBs, in this case, can be measured as corresponding to the risk amount of 1.7 contracts of JGB Futures.

[14] See Tuckman and Serrat (2011) for the definition of BPV and DV01.
[15] Note that this value is based on the calculation as of February 2020, and BPV and other values may change depending on the coupon rate of 20-year JGBs and other factors.

6. The Repo Market and an Implied Repo Rate

6.1 What is the Repo Market?[16]

Regarding any investment decision concerning JGBs, it is vital to consider the funding costs (or the "repo costs" mentioned above) of delivering the JGBs. Individual investors are generally unaware of funding costs as they invest within their holding cash amount. Funding costs, however, become essential to financial institutions and institutional investors; large institutional investors such as banks and insurance companies receive funds from depositors or insurance holders, respectively, and pay the funding costs in a manner of deposit rates. In the case of JGB traders, they also need to raise funds for doing their market-making activities.

One of the essential features of JGB trading is that JGB traders can utilize JGBs as collateral for funding. Using JGBs as collateral has the added advantage of reducing financing costs thanks to the high reliability of the JGBs as a financial instrument. This type of financing, funding with collateral, is called a Repurchase Agreement ("Repo"), and its financing cost at a particular time is called the repo cost or the repo rate.[17]

6.2 GC Repo and SC Repo transactions

Repo transactions are divided into two categories, depending on the needs of the bonds to be delivered: General Collateral (GC) repo and Special Collateral (SC) repo transactions. In the case of GC repo transactions, JGBs are used for funding and investment as collateral in general, without particular consideration of the individuality of each issued bond. On the other hand, SC repo transactions are used when a specific government bond (e.g., 7-year JGBs) is desired, such as when an investor wants to leverage the price difference between the cash-JGB underlying futures and JGB Futures contract.[18] In terms of repo rates, one of the significant differences between GC and SC repo is that the supply-demand balance of the entire repo market determines the GC rate (GC repo rate), whereas the supply-demand balance of the specific individual securities determines the SC rate (SC repo rate). In this context, a spread between GC and SC rates is also used to figure out the supply-demand condition of individual securities. To break down

[16] This section summarizes repo markets related to JGB Futures for readers' further understanding. Readers who wish to learn more about repos should refer to Repo Trading Research (2001), Inamura and Baba (2002), Totan Research (2019), Sasamoto et al. (2020), and others. In addition, the Bank of Japan's Tokyo Short-Term Money Market Survey is useful for understanding repo trends in Japan.
[17] A repo rate is a concept that converts the repo costs into an annualized yield.
[18] As this paper focuses on basis trading in the JGB market, the examples of SC repo transactions here illustrate 7-year JGBs in arbitrage perspectives. As noted by Totan Research (2019), however, many SC market transactions happen according to demands to meet particular cases arising from the market-making process by brokerage firms' traders.

the market share, roughly 70% of recorded transactions were GC repo, and 30% were SC repo as of 2019.[19]

Figure 7: GC repo funding at the time of the JGB issuance

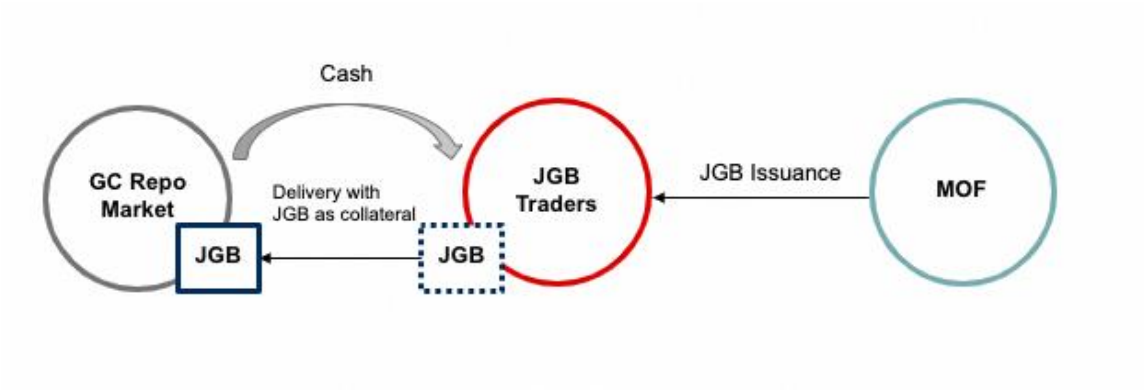
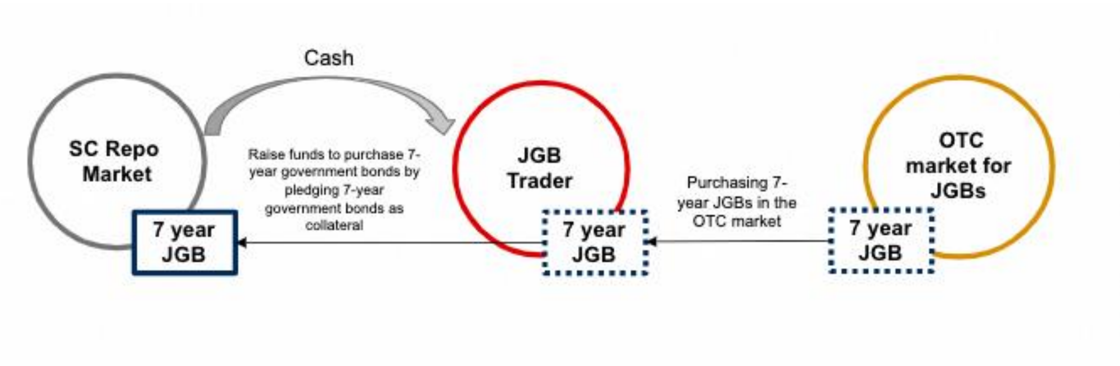


Figure 7 represents how JGB traders use GC repo transactions to fund their bids in JGB auctions.[20] JGB traders raise funds at the time of JGB auctions by pledging JGBs as collateral in the repo market, typically through very short-term funding schemes such as overnight loans.[21] The GC repo rate in this example becomes funding costs for JGB traders.

Next, how do JGB traders finance their funding when they try to trade arbitrage transactions between 7-year JGBs and JGB Futures, which means a basis trade described in the previous sections? To start the arbitrage transactions, a JGB trader needs to secure funds to purchase 7-year JGBs, and the fund is generally financed via the SC repo market. Figure 8 illustrates this transaction, and the SC repo rate in this example becomes funding costs. In the previous sections, we defined a net basis as "(Cash JGB price – Carrying cost) – JGB Futures price × CF," and when calculating the carrying cost (Coupon income – Repo cost), the SC repo rate is used as the Repo cost.[22]

[19] Refer to "Trends in the Japanese Short-Term Money Market - Results of the Tokyo Survey of Short-Term Money Markets (August 2007)" by the BOJ.
[20] This section shows an example of a financing case using the GC repo, but the SC repo can also be used for financing purposes as traders hold specific JGBs in inventory issued at the government bond auction.
[21] For example, if a bidder wins JGBs for JPY 100 billion at auction, the bidder must transfer the payment for settlement to the MOF the next business day of the auction, as the JGB transactions are settled on the T+1 cycle. The bidder is also able to raise funds through the repo market using the JGBs out of the new inventory of JPY 100 billion as collateral. As the settlement cycle of the repo market is also on a T+1 basis, the bidder is available to raise funds using successful JGBs by the next day of the auction and fill the payment to the MOF. Note that in some cases, such as the 2-year government note auction, the settlement cycle is different from the above example. In addition, JGB traders do not always hold all the JGBs won at the auction as inventory and usually liquidate the inventory by selling them to investors who give prior orders to the traders before the auction.
[22] In the case of the arbitrage transactions between JGB Futures and cash JGBs, if the delivery date of the futures contracts is two months later, a trader needs to raise funds for two months to purchase 7-year JGBs. In the practice of the repo market, as short-term funding, such as overnight loans, is more liquid, there are cases where a trader keeps rolling over the short-term positions every time the maturity date comes.

Figure 8: SC repo funding using 7-year JGBs (CTD)



6.3 Implied Repo Rate

The challenge of the arbitrage transactions mentioned above is that if the delivery date is far away, the fair rate of SC repo becomes difficult to observe due to its lower market liquidity. We can hear a fair rate of SC repo by broker-dealers when it comes to overnight transactions, but it is more difficult to obtain a fair value of a longer-term SC repo rate. [23] On the other hand, as all parameters except for a repo rate (e.g., cash JGB and JGB Futures prices) are easy to get when calculating net basis, a repo rate is measured through the reverse calculation of the formula if we assume arbitrage conditions work between JGB Futures and 7-year JGBs. The obtained rate is called the Implied Repo Rate (IRR). The beneficial aspect of the IRR is that by assuming efficient arbitrage conditions, the SC repo rate, which is usually difficult to obtain when the delivery date is far into the future, is figured out.

In calculating the IRR, the SC repo rate is extracted, assuming that the net basis is zero. When the net basis is zero, we can obtain the equation below:

$$\text{Cash JGB price} + (\text{Repo cost} - \text{Coupon Interest income}) - \text{JGB Futures price} \times \text{CF} = 0$$

This equation can be converted to an equation for repo cost in the following form:

$$\text{Repo cost} = \text{JGB Futures price} \times \text{CF} + \text{Coupon Interest income} - \text{Cash JGB price}$$

The IRR is the formula which converts this into an annualized return form.[24]

[23] Call loan brokers are financial institutions that primarily act as intermediaries for short-term lending and borrowing. For more information, refer to Totan Research (2019).
[24] This formula is based on Burghardt and Belton (2005) and Choudhry (2006); Bloomberg uses a definitional formula

$$IRR = \left[\frac{(JGB\ Futures\ price\ \times CF + Coupon\ interest - Cash\ JGB\ price)}{Cash\ JGB\ price}\right] \times \left(\frac{360}{Days\ to\ Delivery}\right)$$

The most significant advantage of using the IRR is that it measures the efficiency of the cash and futures arbitrage. In calculating the IRR, arbitrage relationships between cash JGBs and JGB Futures contracts are assumed (i.e., the net basis is supposed to be zero), but this arbitrage condition does not always realize in the real market. Thus, the effectiveness of arbitrage conditions between cash JGBs and JGB Futures can be assessed by measuring whether the current IRR level is justified by comparing the rate with other repo rates, such as the highly liquid short-term SC repo rate and the term structure of the GC repo rate. As such, in the JGB market, confirming the level of IRR is essential to understand to what extent the JGB Futures prices having the most extensive market liquidity, are reflected in the actual cash JGB prices.

6.4 Implied Repo Rate Calculation Examples and their Interpretation

Table 2: Deliverable JGBs for the December-2019 contracts of JGB Futures

Deliverable JGBs	Price	Years to Maturity	CF	Gross Basis	Net Basis	IRR （%）
10-Year JGB ♯345	102.691	7.14	0.6668	0.043	0.012	-0.21
10-Year JGB ♯346	102.786	7.39	0.6572	1.606	1.575	-11.31
10-Year JGB ♯347	102.883	7.64	0.6478	3.150	3.119	-22.24
10-Year JGB ♯348	102.939	7.89	0.6386	4.631	4.600	-32.74
10-Year JGB ♯349	102.948	8.14	0.6294	6.045	6.014	-42.75
10-Year JGB ♯350	102.907	8.39	0.6205	7.388	7.357	-52.31
10-Year JGB ♯351	102.861	8.64	0.6116	8.705	8.674	-61.66
10-Year JGB ♯352	102.806	8.90	0.6029	9.994	9.963	-70.86
10-Year JGB ♯353	102.743	9.15	0.5943	11.255	11.224	-79.84
10-Year JGB ♯354	102.671	9.39	0.5858	12.488	12.457	-88.68
10-Year JGB ♯355	102.593	9.64	0.5774	13.695	13.664	-97.32

Table 2 shows the list of a deliverable JGB basket for the December-2019 contracts of JGB Futures as of October 31, 2019, alongside their respective gross basis, net basis, and IRR.[25] In this case, the delivery date of the December contracts is December 20, 2019, so the remaining period to the delivery date is less than two months. Although the SC repo rate corresponding to the exact period until the delivery date should be

that explicitly addresses elapsed interest and other factors.

[25] The "JBA Comdty DLV" function on a Bloomberg terminal provides a list of CF, gross basis, net basis, and IRR by issued JGBs.

used for calculating the precise net basis, to simplify the calculation, the 3-month Tokyo repo rate of -0.123%[26] is used to obtain the net basis in Table 2. The JGBs closest to 7 years remaining term to maturity (10-year coupon bearing JGB (#345)) in the table has the smallest net basis and the largest IRR, indicating that the JGBs are the CTD. (We can also confirm in Table 2 that the CF for JGBs with a shorter term to maturity has a larger number). Market participants also use the largest IRR to judge which JGBs become the CTD. This is a snapshot of market data on a specific day, but 7-year JGBs tend to become the CTD in the JGB Futures market even on other days.

In Table 2, the IRR for 10-year JGB #345, the CTD, is -0.206%. As mentioned above, this indicates that if the net basis were zero, the SC repo rate for 10-year JGB #345 would be -0.206% up until the delivery date. The 1-month GC repo rate on this day was -0.080%, and the 3-month repo rate was -0.123%. The specific SC rate is not used here, but market participants can take the market rates from broker-dealers. If the IRR is judged as a fair rate compared with these market rates, we can interpret that the arbitrage between cash JGBs and JGB Futures works well.

Suppose that, after assessing various factors comprehensively, -0.1% of the SC repo rate until the delivery date of cash JGBs is considered reasonable. In this case, the IRR of -0.206% implies that the arbitrage relationship between the cash JGBs and the JGB Futures contracts is inefficient. If the actual SC repo rate is -0.1%, arbitrage opportunities exist by taking a short basis position as the net basis is positive (the net basis becomes 0.015 in this case.) [27]

When calculating net basis and IRR, we should note that the close prices and other statistical numbers of Japan Bond Trading Co., Ltd. (Broker's Broker or BB) are based on the quotes at 3:00 p.m., which may differ from the actual traded prices on the day.[28] From a practical perspective, since the order books of the Broker's Broker record the JGB Futures prices at the time when the CTD is traded, market participants can calculate the IRR using the CTD price and JGB Futures price recorded in the order book of the Broker's Broker.

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[26] The Tokyo Repo Rate is the GC repo rate published by the Japan Securities Dealers Association and is the representative GC repo rate in the yen interest rate markets (its official name is the “Tokyo Repo Rate (Reference Counterparty Average)”). In practice, major Japanese financial institutions submit their reference rates as of 11:00 a.m., and the Rate is calculated by a trimmed arithmetic mean that excludes the highest and lowest 15% equivalents for each covered period. In addition to the 1-month and 3-month rates described in this report, other term rates, including overnight and 1-week ahead, are also published.

[27] Conversely, if the SC repo rate takes a value lower than -0.206% (e.g., -0.3%), the net basis takes a negative value.

[28] In the case of close prices at the BB, the minimum tick size is 0.005% for long-term JGBs, and this fixed tick size may cause a gap with actual traded rates. This could make the net basis deviate from zero.

7. Conclusion

This paper described the basics of futures contracts, focusing on JGB Futures. As emphasized, the essential feature of a futures contract is that it is traded on an exchange. Since JGB Futures are the most liquid market in the JPY rate market, market participants normally check the JGB Futures price to evaluate the fair values of the JPY rate market. Through arbitrage transactions between JGB Futures and 7-year JGBs, the price information of JGB Futures spills over to the cash-JGB market. This is why the JGB Futures market holds a prominent position in the JPY rate market.

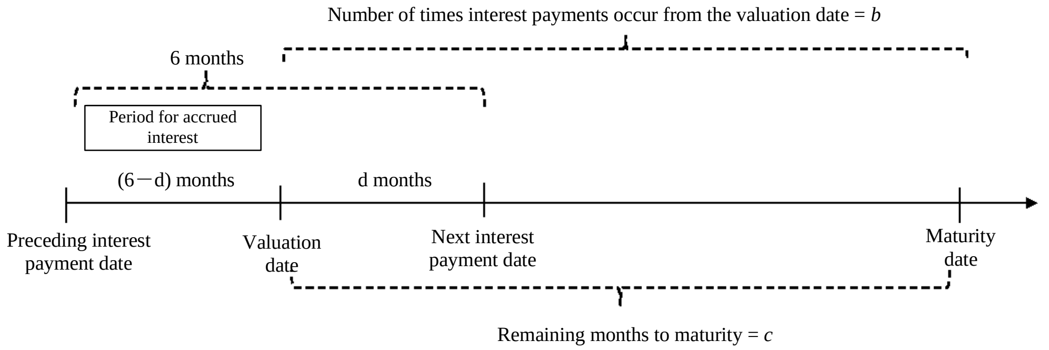
Appendix[29]

Let V^i be the value of deliverable issue when valued on a 6% flat curve (where *i* refers to individual delivery stocks). When *DF* represents the discount factor from the next interest payment date to the valuation date, *FD* is as follows.

$$DF_1 = \frac{1}{(1+0.06/2)^{c/6-(b-1)}}$$

$c/6-(b-1)$ represents the interval on a 6-month basis between the delivery date and the next interest payment day.

Figure 9: Flow of cash flow and accrued interest when deriving the conversion factor



Letting V_1^i represent the price on the next interest payment date in Figure 8, the following holds:

$$V_1^i = \frac{a}{2}\sum_{n=1}^{b}\frac{1}{(1+0.06/2)^{n-1}} + \frac{100}{(1+0.06/2)^{b-1}} = \frac{a/2}{0.06/2}\frac{(1+0.06/2)^{b}-1}{(1+0.06/2)^{b-1}} + \frac{100}{(1+0.06/2)^{b-1}}$$

where $\frac{a}{2}$ represents the amount of interest paid (per JPY 100). Also, based on the definition in Figure 8, the accrued interest is $\frac{a}{2}\times\frac{(6-d)}{6}$.

Therefore, as of the time of evaluation in Figure 8, the price of the deliverable JGB is calculated as follows:

[29] We express our appreciation to Hajime Fujiwara who supported the preparation of this supplementary discussion.

$$
\begin{aligned}
\mathrm{V}^{i} &= DF_1 \times \mathrm{V}_1^{i} - \tfrac{a}{2} \times \tfrac{(6-d)}{6} \\
&= \frac{1}{(1+0.06/2)^{\frac{c}{6}-(b-1)}} \times \left(\frac{\frac{a}{2}}{\frac{0.06}{2}} \frac{(1+0.06/2)^{b}-1}{(1+0.06/2)^{b-1}} + \frac{100}{(1+0.06/2)^{b-1}} \right) - \frac{a}{2} \times \frac{(6-d)}{6} \\
&= \frac{\frac{a}{0.06}\left((1+0.06/2)^{b}-1\right)+100}{(1+0.06/2)^{c/6}} - \frac{a}{2} \times \frac{(6-d)}{6}
\end{aligned}
$$

Therefore, the conversion factor (CF^{i}) is calculated as follows:

$$
\mathrm{CF}^{i} = \frac{\mathrm{V}^{i}}{100} = \frac{\frac{a}{0.06}\left((1+0.06/2)^{b}-1\right)+100}{(1+0.06/2)^{c/6} \times 100} - \frac{a(6-d)}{1200}
$$

References

[1]. Hull, J. 2017. Options, Futures, and Other Derivatives (10th Edition). Pearson

[2]. Tuckman, B., Serrat, A. 2011. Fixed Income Securities: Tools for Today's Markets. Wiley.