Liquidity and Arbitrage Cost between ETF and Stocks using Agent-Based Model

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JPX Working Papers URL:
https://www.jpx.co.jp/english/corporate/research-study/working-paper/index.html
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ETF and Liquidity

Exchange-Traded Fund (ETF)

☑ a mutual fund that invests in a diversified portfolio of many stocks or bonds
☑ listed and traded at a stock exchange

ETFs have been widely spread to individual investors as an easy way to diversify their investments.

Some ETFs, however, have not been traded with enough volume to discover an adequate price, making them difficult for individual investors to trade.
An ETF is exchangeable with all stocks held by the ETF.

When the price of the ETF and the total value of the stocks held by the ETF differ, a trader can buy the cheaper asset, exchange, sell the more expensive asset, and thus earn a profit from the price difference.
The questions, however, of how liquidity changes depending on arbitrage trading costs and of what the mechanism is remain to be answered.

To increase the liquidity of low-liquidity ETFs, in 2018 the Tokyo Stock Exchange introduced a market-making incentive scheme, in which designated market makers always place orders in return for incentives such as lower fees [JPX 17].
Empirical Studies

- cannot be conducted to investigate situations that have never occurred in actual financial markets
- cannot be conducted to isolate the direct effect on liquidity because so many factors affect price formation and liquidity in actual markets

Therefore, in this study,

I expanded the artificial market model of [Mizuta 13] to include three risk assets, two stocks and an ETF. I also added an arbitrage agent to perform arbitrage trading among these risk assets.

I then investigated the relationship between the liquidity of an ETF and the trading costs.
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ETF & Stocks

ETF (one share) = Stock 1 (one share) + Stock 2 (one share)

ETF

fundamental price 20000

Number of Orders = 1/10 for a Stock

Normal Agent places orders with 10% probability to investigate for low liquidity

stock 1

fundamental price 10000

stock 2

fundamental price 10000
For each risk asset, the model includes 1000 normal agents (NAs) that trade only that risk asset, giving a total of 3000 NAs.
Normal Agent (NA)

j: agent number (1,000 agents)
ordering in number order

t: tick time

Expected Return of each NA

\[ r_{e,j}^t = \frac{1}{\sum_i w_{i,j}} \left( w_{1,j} \log \left( \frac{P_f}{P_t} \right) + w_{2,j} r_{h,j}^t + w_{3,j} \epsilon_j^t \right) \]

Parameters for agents

- \( w_{i,j} \) and \( \tau_j \)
  Random of Uniform Distribution
  - \( w_{i,j} \): i=1,3: 0~1
  - \( w_{i,j} \): i=2: 0~10
  - \( \tau_j \): 0~10000

Historical Return

\[ r_{h,j}^t = \log\left( \frac{P_t}{P_{t-\tau_j}} \right) \]

Technical

Expected Price of each NA

\[ P_{e,j}^t = P_t \exp(r_{e,j}^t) \]

Fundamental

- \( P_f \) Fundamental Price
  - 10000 = constant
- \( P_t \) Market Price at t

noise

\( \epsilon_j^t \)
Random of Normal Distribution
Average=0
\( \sigma = 3\% \)
To replicate many waiting limit orders, order price is scattered around expected price.
Arbitrage Agent (AA) (1/3)

The AA can always place, change, or cancel orders.
When Highest buy-order prices (HBPs) consist

HBP for ETF + Cost < HBP for stocks 1 + HBP for stocks 2

AA first places an order to buy one share at 19900 and then waits.

ETF (one share) = Stock 1 (one share) + Stock 2 (one share)

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<th>buy</th>
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<th>sell</th>
<th>price</th>
<th>buy</th>
<th>stock 2</th>
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Includes the required profit, when the price difference of risk assets is over c, the AA can do an arbitrage trade.
Once the order is matched and the AA buys ETF, it exchanges the ETF share for stocks 1 and 2 and then sells them each at 10000. AA earns a profit of 100 from the price difference, \( 10000 \text{ (Stock 1)} + 10000 \text{ (Stock 2)} - 19900 \text{ (ETF)} = 100 \)

Of course, the AA can also earn a profit in the opposite case, by first selling borrowed ETF at a higher price, buying the stocks at lower prices, exchanging the stocks for ETF, and returning the ETF, again earning the price difference as a profit.
Market price differential rate = Mid price of ETF/Sum of Mid prices of stocks - 1

Lower cost meant more trading volume and a lower price differential.

The price differential sharply changed when the cost was near 0.1%, similar to the volatility.

Whether the cost is higher or lower than the volatility seems to indicate a very important boundary.
Relationship between Cost and Volatility

AA can make an arbitrage trade only when the red dashed line is above the black solid line plus the cost.

Prices of each risk asset fluctuate in their volatility

Volatility > Cost

AA has more chances for arbitrage trades
With lower cost, the ETF market became more efficient, but that of stock 1 did not change.

The reason why the ETF market becomes more efficient is NOT because it gains efficiency from the stock 1 market.
The depth for ETF also sharply changed when the cost was near 0.1%. On the other hand, the depth for stock 1 had the opposite tendency.
Lower cost meant higher trading volume for both.

Lower cost makes the depth for stock 1 thinner and the trading volume larger, because orders for arbitrage trades and waiting orders for stock 1 are matched.
Case with more Liquidity for ETF (Cost=0)

(Actually, I fixed the ETF order ratio to $k=0.1$)

Larger ETF order ratio meant thinner depth and more trading volume. More ETF orders caused more matching of arbitrage trade orders and waiting orders for stock 1.
As ETF order ratio increased, market price differential ratio increased.

Even though more arbitrage trades occurred because of the larger ETF order ratio, the market price differential rate did not improve.
(1) Introduction
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I expanded the artificial market model of [Mizuta 13] to include three risk assets, denoted as stock 1, stock 2, and ETF, along with an arbitrage agent (AA) that could perform arbitrage trades among these risk assets. I then investigated the relationship between the liquidity of ETF and the trading costs.

My results showed that, because the prices of each risk asset fluctuate in their volatility, when the volatility is sufficiently greater than the cost, the AA has more chances to make arbitrage trades. As the AA trades more, the market price differential becomes lower.

In addition, lower cost means a thicker depth of waiting trades for ETF, whereas the depth tendency of a stock is the opposite. Furthermore, lower cost increases the trading volume of both. Lower cost makes the depth thinner and the trading volume greater for a stock because the orders for arbitrage trades and the waiting orders for the stock are matched.
Real financial markets, however, include traders who place more orders when the trading volume increases. My model did not implement this behavior. It is possible that lower cost would increase both the depth and the trading volume with such behavior. This remains for a future work.

I also investigated the case with more liquidity for ETF and found that it makes the market price differential larger. Even though more arbitrage trades occur because of the larger ETF liquidity, the market price differential rate does not improve.

This result implies that, when the trading volume of ETF increases to near that of a stock, improving the market price differential is more difficult through arbitrage trades like those modeled in this study. This suggests that other ways are needed to improve the market price differential in such cases.
References

-- market-making incentive scheme --


-- previous model --